Analysis of Algorithms - Quiz II

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1 Instructions

1. The quiz is to be returned by $9:00$ *am.* in class.
2. Each question is worth 3 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. **Divide-And-Conquer:** Use Strassen's matrix mutiplication algorithm to multiply

$$\mathbf{X} = \begin{bmatrix} 3 & 2 \\ 4 & 8 \end{bmatrix} \text{ and } \mathbf{Y} = \begin{bmatrix} 1 & 5 \\ 9 & 6 \end{bmatrix}.$$

2. **Dynamic Programming:** Assume that you are given a chain of matrices $\langle A_1\ A_2\ A_3\ A_4 \rangle$, with dimensions 2×5, 5×4, 4×2 and 2×4 respectively. Compute the optimal number of multiplications required to calculate the chain product.

3. **Shortest Paths:** The matrix $\mathbf{W}$ represents the adjacency matrix of a 4-vertex graph.

$$\mathbf{W} = \begin{bmatrix} 0 & 2 & -3 & \infty \\ \infty & 0 & -2 & 2 \\ \infty & 2 & 0 & 1 \\ \infty & \infty & \infty & 0 \end{bmatrix}$$

   Run the "matrix-multiplication" algorithm on this graph to compute the All-Pairs shortest paths. Recall that in this algorithm, we increase the number of edges on the shortest path between a pair of vertices from 0 to $(n-1)$, on an n-vertex graph. You are required to show all the intermediate matrices.

4. **Shortest Paths:** In class, we discussed how the Floyd-Warshall algorithm is used to compute the length of the shortest path between all vertex-pairs of the input graph; the algorithm can effect this computation *only* if there are no negative cost cycles in the graph. How would you use the Floyd-Warshall approach to declare that the input graph has a negative cost cycle?

5. **String Algorithms:** Draw a compressed trie for the following set of strings:

$$\{abab, baba, ccccc, bbaaa, caa, bbaacc, cbcc, cbca\}.$$