Network Optimization - Quiz

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1 Instructions

1. You are requested to turn in the quiz by Monday, October 13.
2. Based on performance, a maximum of five bonus points will be awarded.

2 Problems

1. Solve the following program using Simplex:

$$\begin{aligned} \max z &= 4x_1 + 3x_2 \\ \text{subject to} & \\ 2x_1 + x_2 &\leq 8 \\ x_1 + x_2 &\leq 5 \\ x_1 - x_2 &\leq 4 \\ x_1 &\leq 4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

2. Solve the following linear program:

$$\begin{aligned} \max z &= 3x_1 + 10x_2 + 5x_5 + 11x_4 + 7x_5 + 14x_6 \\ \text{subject to} & \\ x_1 + 7x_2 + 3x_3 + 4x_4 + 2x_5 + 5x_6 &= 42 \\ x_i &\geq 0 \; i = 1, 2, \ldots 6 \end{aligned}$$

   In general, how would you check whether a singly constrained problem is infeasible? Unbounded? Can you suggest a non-Simplex approach to solving this problem?

3. Consider the following linear program:

$$\begin{aligned} \max z &= x_1 + 2x_2 - 9x_3 + 8x_4 - 36x_5 \\ \text{subject to} & \\ 2x_2 - x_3 + x_4 - 3x_5 &\leq 40 \\ x_1 - x_2 + 2x_4 - 2x_5 &\leq 10 \\ x_i &\geq 0, \; \forall i = 1, 2, \ldots, 5 \end{aligned}$$

   (a) Write down the dual and solve it graphically.

(b) Using the concept of complementary slackness compute the optimal primal solution.

4. Consider the following linear program:

$$\begin{aligned} \max z &= x_1 + 2x_2 - x_3 \\ \text{subject to} & \\ -3x_1 + x_2 + 2x_3 &= 16 \\ 2x_1 + 4x_2 + 3x_3 &\geq 20 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \\ x_3 \text{ unrestricted} & \end{aligned}$$

(a) Write the dual of the above problem without transforming it.

(b) Transform the given problem into the canonical form and then write the dual. Verify that the two dual formulations are equivalent.