Report

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1 QLI

Just like we can reduce quantified 3CNF formulas to QLI by adding an extra quantifier alternation we can similarly reduce quantified 3DNF formulas to QLI.

Say we start with a Q3DNF formula with the quantifier string $\forall\exists\forall$. We can reduce this to a $\forall\exists\forall\exists$ QLI as follows.

For each existentially quantified variable x_i in the original Q3DNF we add the existentially quantified variable x_i.

For each universally quantified variable y_i in the original Q3DNF we add the universally quantified variable y_i and the existentially quantified variable x_{y_i}.

We also add the universally quantified variable z to represent the final disjunction.

We let F represent the right hand side of the implication in the QLI and E the left.

For each x_i we add the constraints $x_i \leq 1$ and $x_i \geq 0$ to F.

For each y_i we add the constraints $y_i \leq 1$ and $y_i \geq 0$ to E, and the constraints $x_{y_i} \leq 1$, $x_{y_i} \leq 2 \cdot y_i$, $x_{y_i} \geq 0$, and $x_{y_i} \geq 2 \cdot y_i - 1$ to F.

For each conjunction of constraints we do the following:

1. If the conjunction is $x_i \wedge y_j \wedge x_k$ we add the constraint $x_i + x_{y_j} + x_k \leq z$ to E.
2. If the conjunction is $\neg x_i \wedge y_j \wedge x_k$ we add the constraint $(1 - x_i) + x_{y_j} + x_k \leq z$ to E.
3. If the conjunction is $\neg x_i \wedge \neg y_j \wedge x_k$ we add the constraint $(1 - x_i) + (1 - x_{y_j}) + x_k \leq z$ to E.
4. If the conjunction is $\neg x_i \wedge \neg y_j \wedge \neg x_k$ we add the constraint $(1 - x_i) + (1 - x_{y_j}) + (1 - x_k) \leq z$ to E.

Finally we add the constraint $z \geq 3$ to F.

Thus the Q3DNF system $\forall y_1 \forall y_2 \exists x_1 \exists x_2 \forall y_3 \ \ (y_1 \wedge x_1 \wedge \neg y_2) \vee (y_3 \wedge \neg x_2 \wedge y_2)$ becomes the QLI.

$$\begin{array}{rcl}
\forall y_1 \forall y_2 \exists x_{y_1} \exists x_{y_2} \exists x_1 \exists x_2 \forall y_3 \forall z \exists x_{y_3} & & \\
y_1 \leq 1, y_1 \geq 0 & & x_{y_1} \leq 1, x_{y_1} \leq 2 \cdot y_1, x_{y_1} \geq 0, x_{y_1} \geq 2 \cdot y_1 - 1 \\
y_2 \leq 1, y_2 \geq 0 & & x_{y_2} \leq 1, x_{y_2} \leq 2 \cdot y_1, x_{y_2} \geq 0, x_{y_2} \geq 2 \cdot y_1 - 1 \\
y_3 \leq 1, y_3 \geq 0 & \rightarrow & x_{y_3} \leq 1, x_{y_3} \leq 2 \cdot y_1, x_{y_3} \geq 0, x_{y_3} \geq 2 \cdot y_1 - 1 \\
x_{y_1} + x_1 + (1 - x_{y_2}) \leq z, x_{y_3} + (1 - x_2) + x_{y_2} \leq z & & z \geq 3
\end{array}$$

This reduction allows us to make the following statement on the complexity of certain types of QLI.

Theorem 1.1 *A* $< \exists, 2 \cdot k, B^{2 \cdot k+1} >$ *QLI is* $\Sigma_P^{2 \cdot k}$*-hard*

Theorem 1.2 *A* $< \forall, 2 \cdot k+1, B^{2 \cdot k+2} >$ *QLI is* $\Pi_P^{2 \cdot k+1}$*-hard*

Thus this reduction allows us to cover the other half of the complexity classes of the PH with differing types of QLI. It also lets us see that QLIs ending with existential quantifiers have differing complexities from those ending in universal quantifiers.