Report

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1 QLI

Just like we can reduce quantified 3CNF formulas to QLI by adding an extra quantifier alternation we can similarly reduce quantified 3DNF formulas to QLI.

Say we start with a Q3DNF formula with the quantifier string $\forall\exists\forall$. We can reduce this to a $\forall\exists\forall\exists$ QLI as follows.

For each existentially quantified variable x_i in the original Q3DNF we add the existentially quantified variable x_i.

For each universally quantified variable y_i in the original Q3DNF we add the universally quantified variable y_i and the existentially quantified variable x_{y_i}.

We also add the universally quantified variable z to represent the final disjunction.

We let F represent the right hand side of the implication in the QLI and E the left.

For each x_i we add the constraints $x_i \leq 1$ and $x_i \geq 0$ to F.

For each y_i we add the constraints $y_i \leq 1$ and $y_i \geq 0$ to E, and the constraints $x_{y_i} \leq 1$, $x_{y_i} \leq 2 \cdot y_i$, $x_{y_i} \geq 0$, and $x_{y_i} \geq 2 \cdot y_i - 1$ to F.

For each conjunction of constraints we do the following:

1. If the conjunction is $x_i \wedge y_j \wedge x_k$ we add the constraint $x_i + x_{y_j} + x_k \leq z$ to E.
2. If the conjunction is $\neg x_i \wedge y_j \wedge x_k$ we add the constraint $(1 - x_i) + x_{y_j} + x_k \leq z$ to E.
3. If the conjunction is $\neg x_i \wedge \neg y_j \wedge x_k$ we add the constraint $(1 - x_i) + (1 - x_{y_j}) + x_k \leq z$ to E.
4. If the conjunction is $\neg x_i \wedge \neg y_j \wedge \neg x_k$ we add the constraint $(1 - x_i) + (1 - x_{y_j}) + (1 - x_k) \leq z$ to E.

Finally we add the constraint $z \geq 3$ to F.

Thus the Q3DNF system $\forall y_1 \forall y_2 \exists x_1 \exists x_2 \forall y_3 \ (y_1 \wedge x_1 \wedge \neg y_2) \vee (y_3 \wedge \neg x_2 \wedge y_2)$ becomes the QLI.

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\begin{array}{rcl}
\forall y_1 \forall y_2 \exists x_{y_1} \exists x_{y_2} \exists x_1 \exists x_2 \forall y_3 \forall z \exists x_{y_3} & & \\
y_1 \leq 1, y_1 \geq 0 & & x_{y_1} \leq 1, x_{y_1} \leq 2 \cdot y_1, x_{y_1} \geq 0, x_{y_1} \geq 2 \cdot y_1 - 1 \\
y_2 \leq 1, y_2 \geq 0 & & x_{y_2} \leq 1, x_{y_2} \leq 2 \cdot y_1, x_{y_2} \geq 0, x_{y_2} \geq 2 \cdot y_1 - 1 \\
y_3 \leq 1, y_3 \geq 0 & \rightarrow & x_{y_3} \leq 1, x_{y_3} \leq 2 \cdot y_1, x_{y_3} \geq 0, x_{y_3} \geq 2 \cdot y_1 - 1 \\
x_{y_1} + x_1 + (1 - x_{y_2}) \leq z, x_{y_3} + (1 - x_2) + x_{y_2} \leq z & & z \geq 3
\end{array}
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