Formal Logic - Introduction

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Why Logic?

(i) The Law!

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- (ii) Mathematics.

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- (iii) Computer Science

- (i) The Law!
- (ii) Mathematics.
- (iii) Computer Science (Automated Reasoning).

Sample Problem (Propositional Logic)

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Argument

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Sample Problem (Propositional Logic)

Argument

If a Democrat is elected, then taxes will go up.

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Either a Democrat will be elected or the bill will pass.

Argument

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Therefore, if the taxes do not go up, then the bill will pass.

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Questions of Interest

Is the above argument valid?

Argument

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- Is the above argument valid?
- What is a valid argument?

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- What is a valid argument?
- What is an argument?

Argument

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Either a Democrat will be elected or the bill will pass.

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- Is the above argument valid?
- What is a valid argument?
- What is an argument?
- How do I prove the validity of an argument?

Sample Problem (Predicate Logic)

Sample Problem (Predicate Logic)

Argument

Sample Problem (Predicate Logic)

Argument

Every farmer owns a cow.

Sample Problem (Predicate Logic)

Argument

Every farmer owns a cow. No dentist owns a cow.

Sample Problem (Predicate Logic)

Argument

Every farmer owns a cow. No dentist owns a cow. Therefore, no dentist is a farmer.

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- Is the above argument valid?
- What is a valid argument?

Argument

Every farmer owns a cow. No dentist owns a cow. Therefore, no dentist is a farmer.

- Is the above argument valid?
- What is a valid argument?
- What are our tools of inference?

Argument

Every farmer owns a cow. No dentist owns a cow. Therefore, no dentist is a farmer.

Questions of Interest

- Is the above argument valid?
- What is a valid argument?
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Note

Argument

Every farmer owns a cow. No dentist owns a cow. Therefore, no dentist is a farmer.

Questions of Interest

- Is the above argument valid?
- 2 What is a valid argument?
- What are our tools of inference?

Note

More generally, we are interested in deducing new facts, given a set of facts and inferences.

Sample Problem (Number theory)

Sample Problem (Number theory)

Problem

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Sample Problem (Number theory)

Problem

Prove that the sum of the first *n* positive integers is $\frac{n \cdot (n+1)}{2}$.

Sample Problem (Set Theory)

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Problem

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Sample Problem (Set Theory)

Problem

Let A and B denote two sets.

Sample Problem (Set Theory)

Problem

Let *A* and *B* denote two sets. Argue that $(A \cap B) \subseteq A$.

Automated Reasoning

Automated Reasoning

An algorithm

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Automated Reasoning

An algorithm

Consider the following algorithm:

Automated Reasoning

An algorithm

Consider the following algorithm:

```
Function MAX-FIND(A, n)
```

```
1: if (n == 1) then
```

```
2: return(A[1]).
```

```
3: else
```

```
4: return(max(A[n], MAX-FIND(\mathbf{A}, n - 1))).
```

```
5: end if
```

Questions of Interest

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Questions of Interest

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Questions of Interest

• Is the above algorithm correct?

- Is the above algorithm correct?
- 2 What is the definition of correctness?

- Is the above algorithm correct?
- What is the definition of correctness?
- 3 Can you provide a proof of correctness?

- Is the above algorithm correct?
- What is the definition of correctness?
- On you provide a proof of correctness?
- What is a proof?

- Is the above algorithm correct?
- What is the definition of correctness?
- On you provide a proof of correctness?
- What is a proof?
- On you analyze the resources taken by the algorithm?