Predicate Logic (First Order Logic)

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Outline

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1. Quantifiers and Predicates
2. Translation
3. Validity

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Quantifiers

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(iv) The existential quantifier $(\exists x)P(x)$ indicates that property P holds for some x in the domain.

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For instance, $(\exists x)(\forall y)Q(x, y)$ *does not mean the same thing as* $(\forall y)(\exists x)Q(x, y)$.

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(ii) An assignment of a property of the objects in the domain to each predicate in the expression.
(iii) An assignment of a particular object in the domain to each constant symbol in the expression.

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What happens when the domain is the set of all integers?

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The scope of a quantifier is the portion of the predicate formula to which it applies.

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The axioms of the theory will be added to the axioms of Predicate Logic and we will reason using the tools of Predicate Logic.

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(ii) Propositional rules are not sufficient. For instance, you cannot use propositional rules to conclude validity in the Socrates example.

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For instance, from $(\forall x)(\exists y)P(x, y)$, you cannot conclude $(\exists y)P(y, y)$.

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Consider the following proof sequence:

(i) $(\forall x)[H(x) \rightarrow M(x)]$ hypothesis.

(ii) $H(s) \rightarrow M(s)$ (i), ui.

(iii) $H(s)$ hypothesis.

(iv) $M(s)$ (ii), (iii), Modus Ponens.

Example

Prove that the following argument is valid.

UI Example

Example

Let us prove that the following argument is valid, using ui.

$$[(\forall x)[H(x) \rightarrow M(x)] \land H(s)] \rightarrow M(s)$$

Proof

Consider the following proof sequence:

(i) $(\forall x)[H(x) \rightarrow M(x)]$ hypothesis.

(ii) $H(s) \rightarrow M(s)$ (i), ui.

(iii) $H(s)$ hypothesis.

(iv) $M(s)$ (ii), (iii), Modus Ponens.

Example

Prove that the following argument is valid.

$$[(\forall x)[P(x) \rightarrow R(x)] \land (R(y))'] \rightarrow (P(y))'$$

Existential Instantiation

Existential Instantiation

Details

Existential Instantiation

Details

Existential Instantiation

Details

(i) From $(\exists x)P(x)$, you can conclude $P(a)$, where a is a constant symbol not used previously in the proof sequence.

Existential Instantiation

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(i) From $(\exists x)P(x)$, you can conclude $P(a)$, where a is a constant symbol not used previously in the proof sequence.

(ii) Rule is abbreviated as e.i.

Existential Instantiation

Details

(i) From $(\exists x)P(x)$, you can conclude $P(a)$, where a is a constant symbol not used previously in the proof sequence.

(ii) Rule is abbreviated as e.i. (or ei).

Existential Instantiation

Details

(i) From $(\exists x)P(x)$, you can conclude $P(a)$, where a is a constant symbol not used previously in the proof sequence.

(ii) Rule is abbreviated as e.i. (or ei).

(iii) Must be the first rule that introduces a.

EI example

EI example

Example

EI example

Example

Show that $[(\forall x)[P(x) \rightarrow Q(x)] \land (\exists y)P(y)] \rightarrow Q(a)$ is valid.

EI example

Example

Show that $[(\forall x)[P(x) \rightarrow Q(x)] \land (\exists y)P(y)] \rightarrow Q(a)$ is valid.

Proof

EI example

Example

Show that $[(\forall x)[P(x) \rightarrow Q(x)] \wedge (\exists y)P(y)] \rightarrow Q(a)$ is valid.

Proof

Consider the following proof sequence.

EI example

Example

Show that $[(\forall x)[P(x) \rightarrow Q(x)] \wedge (\exists y)P(y)] \rightarrow Q(a)$ is valid.

Proof

Consider the following proof sequence.

(i) $(\exists y)P(y)$ hypothesis.

EI example

Example

Show that $[(\forall x)[P(x) \rightarrow Q(x)] \land (\exists y)P(y)] \rightarrow Q(a)$ is valid.

Proof

Consider the following proof sequence.

(i) $(\exists y)P(y)$ hypothesis.

(ii) $P(a)$ (i), ei.

EI example

Example

Show that $[(\forall x)[P(x) \rightarrow Q(x)] \wedge (\exists y)P(y)] \rightarrow Q(a)$ is valid.

Proof

Consider the following proof sequence.

(i) $(\exists y)P(y)$ hypothesis.

(ii) $P(a)$ (i), ei.

(iii) $(\forall x)[P(x) \rightarrow Q(x)]$ hypothesis.

EI example

Example

Show that $[(\forall x)[P(x) \rightarrow Q(x)] \wedge (\exists y)P(y)] \rightarrow Q(a)$ is valid.

Proof

Consider the following proof sequence.

(i) $(\exists y)P(y)$ hypothesis.
(ii) $P(a)$ (i), ei.
(iii) $(\forall x)[P(x) \rightarrow Q(x)]$ hypothesis.
(iv) $P(a) \rightarrow Q(a)$ (iii), ui.

EI example

Example

Show that $[(\forall x)[P(x) \rightarrow Q(x)] \land (\exists y)P(y)] \rightarrow Q(a)$ is valid.

Proof

Consider the following proof sequence.

(i) $(\exists y)P(y)$ hypothesis.

(ii) $P(a)$ (i), ei.

(iii) $(\forall x)[P(x) \rightarrow Q(x)]$ hypothesis.

(iv) $P(a) \rightarrow Q(a)$ (iii), ui.

(v) $Q(a)$ (ii), (iv), Modus Ponens.

EI example

Example

Show that $[(\forall x)[P(x) \rightarrow Q(x)] \wedge (\exists y)P(y)] \rightarrow Q(a)$ is valid.

Proof

Consider the following proof sequence.

(i) $(\exists y)P(y)$ hypothesis.

(ii) $P(a)$ (i), ei.

(iii) $(\forall x)[P(x) \rightarrow Q(x)]$ hypothesis.

(iv) $P(a) \rightarrow Q(a)$ (iii), ui.

(v) $Q(a)$ (ii), (iv), Modus Ponens.

Note

EI example

Example

Show that $[(\forall x)[P(x) \rightarrow Q(x)] \wedge (\exists y)P(y)] \rightarrow Q(a)$ is valid.

Proof

Consider the following proof sequence.

(i) $(\exists y)P(y)$ hypothesis.
(ii) $P(a)$ (i), ei.
(iii) $(\forall x)[P(x) \rightarrow Q(x)]$ hypothesis.
(iv) $P(a) \rightarrow Q(a)$ (iii), ui.
(v) $Q(a)$ (ii), (iv), Modus Ponens.

Note

Steps (i)-(ii) and (iii)-(iv) **cannot** *be interchanged.*

Universal Generalization

Universal Generalization

Details

Universal Generalization

Details

(i) From $P(x)$, you can conclude $(\forall x)P(x)$.

Universal Generalization

Details

(i) From $P(x)$, you can conclude $(\forall x)P(x)$.

(ii) Rule is abbreviated as u.g.

Universal Generalization

Details

(i) From $P(x)$, you can conclude $(\forall x)P(x)$.

(ii) Rule is abbreviated as u.g. (or ug).

Universal Generalization

Details

(i) From $P(x)$, you can conclude $(\forall x)P(x)$.
(ii) Rule is abbreviated as u.g. (or ug).
(iii) $P(x)$ has not been deduced from a hypothesis in which x is a free variable.

Universal Generalization

Details

(i) From $P(x)$, you can conclude $(\forall x)P(x)$.

(ii) Rule is abbreviated as u.g. (or ug).

(iii) $P(x)$ has not been deduced from a hypothesis in which x is a free variable.
Also, $P(x)$ has not been deduced using e.i. from any wff, in which x is a free variable.

UG example

UG example

Example

UG example

Example

Show that the following argument is valid.

$$[(\forall x)[P(x) \rightarrow Q(x)] \wedge (\forall x)P(x)] \rightarrow (\forall x)Q(x).$$

UG example

Example

Show that the following argument is valid.

$$[(\forall x)[P(x) \rightarrow Q(x)] \land (\forall x)P(x)] \rightarrow (\forall x)Q(x).$$

Proof

UG example

Example

Show that the following argument is valid.

$$[(\forall x)[P(x) \rightarrow Q(x)] \wedge (\forall x)P(x)] \rightarrow (\forall x)Q(x).$$

Proof

Consider the following proof sequence:

UG example

Example

Show that the following argument is valid.

$$[(\forall x)[P(x) \rightarrow Q(x)] \wedge (\forall x)P(x)] \rightarrow (\forall x)Q(x).$$

Proof

Consider the following proof sequence:

(i) $(\forall x)[P(x) \rightarrow Q(x)]$ hypothesis.

UG example

Example

Show that the following argument is valid.

$$[(\forall x)[P(x) \rightarrow Q(x)] \wedge (\forall x)P(x)] \rightarrow (\forall x)Q(x).$$

Proof

Consider the following proof sequence:

(i) $(\forall x)[P(x) \rightarrow Q(x)]$ hypothesis.

(ii) $P(x) \rightarrow Q(x)$ (i), ui.

UG example

Example

Show that the following argument is valid.

$$[(\forall x)[P(x) \rightarrow Q(x)] \land (\forall x)P(x)] \rightarrow (\forall x)Q(x).$$

Proof

Consider the following proof sequence:

(i) $(\forall x)[P(x) \rightarrow Q(x)]$ hypothesis.

(ii) $P(x) \rightarrow Q(x)$ (i), ui.

(iii) $(\forall x)P(x)$ hypothesis.

UG example

Example

Show that the following argument is valid.

$$[(\forall x)[P(x) \rightarrow Q(x)] \land (\forall x)P(x)] \rightarrow (\forall x)Q(x).$$

Proof

Consider the following proof sequence:

(i) $(\forall x)[P(x) \rightarrow Q(x)]$ hypothesis.

(ii) $P(x) \rightarrow Q(x)$ (i), ui.

(iii) $(\forall x)P(x)$ hypothesis.

(iv) $P(x)$ (iii), ui.

UG example

Example

Show that the following argument is valid.

$$[(\forall x)[P(x) \rightarrow Q(x)] \wedge (\forall x)P(x)] \rightarrow (\forall x)Q(x).$$

Proof

Consider the following proof sequence:

(i) $(\forall x)[P(x) \rightarrow Q(x)]$ hypothesis.

(ii) $P(x) \rightarrow Q(x)$ (i), ui.

(iii) $(\forall x)P(x)$ hypothesis.

(iv) $P(x)$ (iii), ui.

(v) $Q(x)$ (ii), (iv) Modus Ponens.

UG example

Example

Show that the following argument is valid.

$$[(\forall x)[P(x) \rightarrow Q(x)] \land (\forall x)P(x)] \rightarrow (\forall x)Q(x).$$

Proof

Consider the following proof sequence:

(i) $(\forall x)[P(x) \rightarrow Q(x)]$ hypothesis.
(ii) $P(x) \rightarrow Q(x)$ (i), ui.
(iii) $(\forall x)P(x)$ hypothesis.
(iv) $P(x)$ (iii), ui.
(v) $Q(x)$ (ii), (iv) Modus Ponens.
(vi) $(\forall x)Q(x)$ (v), ug. (Neither restriction has been violated.)

Incorrect Usage of UG

Incorrect Usage of UG

Free Variable rule violation

Incorrect Usage of UG

Free Variable rule violation

Consider the following proof sequence:

Incorrect Usage of UG

Free Variable rule violation

Consider the following proof sequence:

(i) $P(x)$ hypothesis.

Incorrect Usage of UG

Free Variable rule violation

Consider the following proof sequence:

(i) $P(x)$ hypothesis.

(ii) $(\forall x)P(x)$ (i), ug.

Incorrect Usage of UG

Free Variable rule violation

Consider the following proof sequence:

(i) $P(x)$ hypothesis.

(ii) $(\forall x)P(x)$ (i), ug.

Is the above use of ug correct?

Incorrect Usage of UG

Free Variable rule violation

Consider the following proof sequence:

(i) $P(x)$ hypothesis.

(ii) $(\forall x)P(x)$ (i), ug.

Is the above use of ug correct? Why not?

Incorrect Usage of UG

Free Variable rule violation

Consider the following proof sequence:

(i) $P(x)$ hypothesis.

(ii) $(\forall x)P(x)$ (i), ug.

Is the above use of ug correct? Why not?

EI rule violation

Incorrect Usage of UG

Free Variable rule violation

Consider the following proof sequence:

(i) $P(x)$ hypothesis.

(ii) $(\forall x)P(x)$ (i), ug.

Is the above use of ug correct? Why not?

EI rule violation

Consider the following proof sequence:

Incorrect Usage of UG

Free Variable rule violation

Consider the following proof sequence:

(i) $P(x)$ hypothesis.

(ii) $(\forall x)P(x)$ (i), ug.

Is the above use of ug correct? Why not?

EI rule violation

Consider the following proof sequence:

(i) $(\forall x)(\exists y)Q(x, y)$ hypothesis.

Incorrect Usage of UG

Free Variable rule violation

Consider the following proof sequence:

(i) $P(x)$ hypothesis.

(ii) $(\forall x)P(x)$ (i), ug.

Is the above use of ug correct? Why not?

EI rule violation

Consider the following proof sequence:

(i) $(\forall x)(\exists y)Q(x, y)$ hypothesis.

(ii) $(\exists y)Q(x, y)$ (i), ui.

Incorrect Usage of UG

Free Variable rule violation

Consider the following proof sequence:

(i) $P(x)$ hypothesis.

(ii) $(\forall x)P(x)$ (i), ug.

Is the above use of ug correct? Why not?

EI rule violation

Consider the following proof sequence:

(i) $(\forall x)(\exists y)Q(x, y)$ hypothesis.

(ii) $(\exists y)Q(x, y)$ (i), ui.

(iii) $Q(x, a)$ (ii), ei.

Incorrect Usage of UG

Free Variable rule violation

Consider the following proof sequence:

(i) $P(x)$ hypothesis.

(ii) $(\forall x)P(x)$ (i), ug.

Is the above use of ug correct? Why not?

EI rule violation

Consider the following proof sequence:

(i) $(\forall x)(\exists y)Q(x, y)$ hypothesis.

(ii) $(\exists y)Q(x, y)$ (i), ui.

(iii) $Q(x, a)$ (ii), ei.

(iv) $(\forall x)Q(x, a)$ (iii), ug.

Incorrect Usage of UG

Free Variable rule violation

Consider the following proof sequence:

(i) $P(x)$ hypothesis.
(ii) $(\forall x)P(x)$ (i), ug.

Is the above use of ug correct? Why not?

EI rule violation

Consider the following proof sequence:

(i) $(\forall x)(\exists y)Q(x, y)$ hypothesis.
(ii) $(\exists y)Q(x, y)$ (i), ui.
(iii) $Q(x, a)$ (ii), ei.
(iv) $(\forall x)Q(x, a)$ (iii), ug.

Is the above use of ug correct?

Incorrect Usage of UG

Free Variable rule violation

Consider the following proof sequence:

(i) $P(x)$ hypothesis.

(ii) $(\forall x)P(x)$ (i), ug.

Is the above use of ug correct? Why not?

EI rule violation

Consider the following proof sequence:

(i) $(\forall x)(\exists y)Q(x, y)$ hypothesis.

(ii) $(\exists y)Q(x, y)$ (i), ui.

(iii) $Q(x, a)$ (ii), ei.

(iv) $(\forall x)Q(x, a)$ (iii), ug.

Is the above use of ug correct? Why not?

Existential Generalization

Existential Generalization

Details

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Details

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Details

(i) From $P(x)$ or $P(a)$, you can conclude $(\exists x)P(x)$.

Existential Generalization

Details

(i) From $P(x)$ or $P(a)$, you can conclude $(\exists x)P(x)$.

(ii) Rule is abbreviated as e.g.

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Existential Generalization

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(i) From $P(x)$ or $P(a)$, you can conclude $(\exists x)P(x)$.

(ii) Rule is abbreviated as e.g. (or eg).

(iii) To go from $P(a)$ to $(\exists x)P(x)$, x must not appear in $P(a)$.

Existential Generalization

Details

(i) From $P(x)$ or $P(a)$, you can conclude $(\exists x)P(x)$.

(ii) Rule is abbreviated as e.g. (or eg).

(iii) To go from $P(a)$ to $(\exists x)P(x)$, x must not appear in $P(a)$.
Otherwise, we could derive $(\exists y)Q(y, y)$ from $Q(a, y)$!

Existential Generalization

Details

(i) From $P(x)$ or $P(a)$, you can conclude $(\exists x)P(x)$.

(ii) Rule is abbreviated as e.g. (or eg).

(iii) To go from $P(a)$ to $(\exists x)P(x)$, x must not appear in $P(a)$.
Otherwise, we could derive $(\exists y)Q(y, y)$ from $Q(a, y)$!
The argument $Q(a, y) \rightarrow (\exists y)Q(y, y)$ is simply not valid. (Why?)

EG example

EG example

Example

EG example

Example

Show that the following argument is valid.

$$(\forall x)P(x) \rightarrow (\exists x)P(x)$$

EG example

Example

Show that the following argument is valid.

$$(\forall x)P(x) \rightarrow (\exists x)P(x)$$

Proof

EG example

Example

Show that the following argument is valid.

$$(\forall x)P(x) \rightarrow (\exists x)P(x)$$

Proof

Consider the following proof sequence:

EG example

Example

Show that the following argument is valid.

$$(\forall x)P(x) \rightarrow (\exists x)P(x)$$

Proof

Consider the following proof sequence:

(i) $(\forall x)P(x)$ hypothesis.

EG example

Example

Show that the following argument is valid.

$$(\forall x)P(x) \rightarrow (\exists x)P(x)$$

Proof

Consider the following proof sequence:

(i) $(\forall x)P(x)$ hypothesis.

(ii) $P(x)$ (i), ui.

EG example

Example

Show that the following argument is valid.

$$(\forall x)P(x) \rightarrow (\exists x)P(x)$$

Proof

Consider the following proof sequence:

(i) $(\forall x)P(x)$ hypothesis.

(ii) $P(x)$ (i), ui.

(iii) $(\exists x)P(x)$ (ii), eg.

Temporary Hypothesis

Temporary Hypothesis

Details

Temporary Hypothesis

Details

Temporary Hypothesis

Details

1. In the midst of a proof, you can assume a suitable wff formula T.

Temporary Hypothesis

Details

1. In the midst of a proof, you can assume a suitable wff formula T.
2. T is called the “temporary hypothesis”.

Temporary Hypothesis

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1. In the midst of a proof, you can assume a suitable wff formula T.
2. T is called the “temporary hypothesis”.
3. If you can deduce the wff W, as a consequence of this assumption, then this means that the wff $T \rightarrow W$ has been deduced from the initial hypotheses.

Temporary Hypothesis

Details

1. In the midst of a proof, you can assume a suitable wff formula T.
2. T is called the "temporary hypothesis".
3. If you can deduce the wff W, as a consequence of this assumption, then this means that the wff $T \rightarrow W$ has been deduced from the initial hypotheses.
4. Observe that this method is a generalization of the Deduction Method to Predicate Logic.

Temporary Hypothesis

Details

1. In the midst of a proof, you can assume a suitable wff formula T.
2. T is called the "temporary hypothesis".
3. If you can deduce the wff W, as a consequence of this assumption, then this means that the wff $T \rightarrow W$ has been deduced from the initial hypotheses.
4. Observe that this method is a generalization of the Deduction Method to Predicate Logic.
5. In the Deduction Method, if the desired conclusion is of the form $P \rightarrow Q$, then we simply assume P and deduce Q.

Temporary Hypothesis

Details

1. In the midst of a proof, you can assume a suitable wff formula T.
2. T is called the “temporary hypothesis”.
3. If you can deduce the wff W, as a consequence of this assumption, then this means that the wff $T \to W$ has been deduced from the initial hypotheses.
4. Observe that this method is a generalization of the Deduction Method to Predicate Logic.
5. In the Deduction Method, if the desired conclusion is of the form $P \to Q$, then we simply assume P and deduce Q.
6. However, this cannot be done, if the conclusion is of the form: $(\forall x)[P(x) \to Q(x)]$ or $(\exists x)[P(x) \to Q(x)]$.

Temporary Hypothesis (example)

Temporary Hypothesis (example)

Example

Temporary Hypothesis (example)

Example

Show that the following formula is valid:

Temporary Hypothesis (example)

Example

Show that the following formula is valid:

$$[P(x) \to (\forall y)Q(x,y)] \to (\forall y)[P(x) \to Q(x,y)].$$

Temporary Hypothesis (example)

Example

Show that the following formula is valid:

$$[P(x) \rightarrow (\forall y)Q(x,y)] \rightarrow (\forall y)[P(x) \rightarrow Q(x,y)].$$

Proof

Temporary Hypothesis (example)

Example

Show that the following formula is valid:

$$[P(x) \rightarrow (\forall y)Q(x, y)] \rightarrow (\forall y)[P(x) \rightarrow Q(x, y)].$$

Proof

Consider the following proof sequence:

Temporary Hypothesis (example)

Example

Show that the following formula is valid:

$$[P(x) \rightarrow (\forall y)Q(x,y)] \rightarrow (\forall y)[P(x) \rightarrow Q(x,y)].$$

Proof

Consider the following proof sequence:

(i) $P(x) \rightarrow (\forall y)Q(x,y)$ hypothesis.

Temporary Hypothesis (example)

Example

Show that the following formula is valid:

$$[P(x) \to (\forall y)Q(x, y)] \to (\forall y)[P(x) \to Q(x, y)].$$

Proof

Consider the following proof sequence:

(i) $P(x) \to (\forall y)Q(x, y)$ hypothesis.

(ii) $P(x)$ temporary hypothesis.

Temporary Hypothesis (example)

Example

Show that the following formula is valid:

$$[P(x) \rightarrow (\forall y)Q(x,y)] \rightarrow (\forall y)[P(x) \rightarrow Q(x,y)].$$

Proof

Consider the following proof sequence:

(i) $P(x) \rightarrow (\forall y)Q(x,y)$ hypothesis.

(ii) $P(x)$ temporary hypothesis.

(iii) $(\forall y)Q(x,y)$ (i), (ii), Modus Ponens.

Temporary Hypothesis (example)

Example

Show that the following formula is valid:

$$[P(x) \rightarrow (\forall y)Q(x,y)] \rightarrow (\forall y)[P(x) \rightarrow Q(x,y)].$$

Proof

Consider the following proof sequence:

(i) $P(x) \rightarrow (\forall y)Q(x,y)$ hypothesis.

(ii) $P(x)$ temporary hypothesis.

(iii) $(\forall y)Q(x,y)$ (i), (ii), Modus Ponens.

(iv) $Q(x,y)$ (iii), ui.

Temporary Hypothesis (example)

Example

Show that the following formula is valid:

$$[P(x) \rightarrow (\forall y)Q(x,y)] \rightarrow (\forall y)[P(x) \rightarrow Q(x,y)].$$

Proof

Consider the following proof sequence:

(i) $P(x) \rightarrow (\forall y)Q(x,y)$ hypothesis.

(ii) $P(x)$ temporary hypothesis.

(iii) $(\forall y)Q(x,y)$ (i), (ii), Modus Ponens.

(iv) $Q(x,y)$ (iii), ui.

(v) $P(x) \rightarrow Q(x,y)$ temporary hypothesis discharged.

Temporary Hypothesis (example)

Example

Show that the following formula is valid:

$$[P(x) \rightarrow (\forall y)Q(x,y)] \rightarrow (\forall y)[P(x) \rightarrow Q(x,y)].$$

Proof

Consider the following proof sequence:

(i) $P(x) \rightarrow (\forall y)Q(x,y)$ hypothesis.

(ii) $P(x)$ temporary hypothesis.

(iii) $(\forall y)Q(x,y)$ (i), (ii), Modus Ponens.

(iv) $Q(x,y)$ (iii), ui.

(v) $P(x) \rightarrow Q(x,y)$ temporary hypothesis discharged.

(vi) $(\forall y)[P(x) \rightarrow Q(x,y)]$ (v), ug.

Principal Techniques of Proving Validity in Predicate Arguments

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Main points of predicate rules

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Main points of predicate rules

(i) Strip off quantifiers.

Principal Techniques of Proving Validity in Predicate Arguments

Main points of predicate rules

(i) Strip off quantifiers.
(ii) Work with separate wffs.

Principal Techniques of Proving Validity in Predicate Arguments

Main points of predicate rules

(i) Strip off quantifiers.
(ii) Work with separate wffs.
(iii) Insert quantifiers as necessary.

Exercises

Exercises

Example

Exercises

Example

Show that the following argument is valid:

$$(\forall x)[P(x) \land Q(x)] \rightarrow (\forall x)P(x) \land (\forall x)Q(x).$$

Exercises

Example

Show that the following argument is valid:

$$(\forall x)[P(x) \wedge Q(x)] \rightarrow (\forall x)P(x) \wedge (\forall x)Q(x).$$

Proof

Exercises

Example

Show that the following argument is valid:

$$(\forall x)[P(x) \land Q(x)] \to (\forall x)P(x) \land (\forall x)Q(x).$$

Proof

Consider the following proof sequence:

Exercises

Example

Show that the following argument is valid:

$$(\forall x)[P(x) \land Q(x)] \rightarrow (\forall x)P(x) \land (\forall x)Q(x).$$

Proof

Consider the following proof sequence:

(i) $(\forall x)[P(x) \land Q(x)]$ hypothesis.

Exercises

Example

Show that the following argument is valid:

$$(\forall x)[P(x) \land Q(x)] \rightarrow (\forall x)P(x) \land (\forall x)Q(x).$$

Proof

Consider the following proof sequence:

(i) $(\forall x)[P(x) \land Q(x)]$ hypothesis.

(ii) $P(x) \land Q(x)$ (i), ui.

Exercises

Example

Show that the following argument is valid:

$$(\forall x)[P(x) \land Q(x)] \rightarrow (\forall x)P(x) \land (\forall x)Q(x).$$

Proof

Consider the following proof sequence:

(i) $(\forall x)[P(x) \land Q(x)]$ hypothesis.

(ii) $P(x) \land Q(x)$ (i), ui.

(iii) $P(x)$ (ii), Simplification.

Exercises

Example

Show that the following argument is valid:

$$(\forall x)[P(x) \land Q(x)] \rightarrow (\forall x)P(x) \land (\forall x)Q(x).$$

Proof

Consider the following proof sequence:

(i) $(\forall x)[P(x) \land Q(x)]$ hypothesis.

(ii) $P(x) \land Q(x)$ (i), ui.

(iii) $P(x)$ (ii), Simplification.

(iv) $(\forall x)P(x)$ (iii), ug.

Exercises

Example

Show that the following argument is valid:

$$(\forall x)[P(x) \land Q(x)] \rightarrow (\forall x)P(x) \land (\forall x)Q(x).$$

Proof

Consider the following proof sequence:

(i) $(\forall x)[P(x) \land Q(x)]$ hypothesis.

(ii) $P(x) \land Q(x)$ (i), ui.

(iii) $P(x)$ (ii), Simplification.

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Exercises

Example

Show that the following argument is valid:

$$(\forall x)[P(x) \land Q(x)] \rightarrow (\forall x)P(x) \land (\forall x)Q(x).$$

Proof

Consider the following proof sequence:

(i) $(\forall x)[P(x) \land Q(x)]$ hypothesis.
(ii) $P(x) \land Q(x)$ (i), ui.
(iii) $P(x)$ (ii), Simplification.
(iv) $(\forall x)P(x)$ (iii), ug.
(v) $Q(x)$ (ii), Simplification.
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Exercises

Example

Show that the following argument is valid:

$$(\forall x)[P(x) \land Q(x)] \rightarrow (\forall x)P(x) \land (\forall x)Q(x).$$

Proof

Consider the following proof sequence:

(i) $(\forall x)[P(x) \land Q(x)]$ hypothesis.

(ii) $P(x) \land Q(x)$ (i), ui.

(iii) $P(x)$ (ii), Simplification.

(iv) $(\forall x)P(x)$ (iii), ug.

(v) $Q(x)$ (ii), Simplification.

(vi) $(\forall x)Q(x)$ (v), ug.

(vii) $(\forall x)P(x) \land (\forall x)Q(x)$ (iv), (vi), Conjunction.

Exercises

Example

Show that the following argument is valid:

$$(\forall x)[P(x) \land Q(x)] \rightarrow (\forall x)P(x) \land (\forall x)Q(x).$$

Proof

Consider the following proof sequence:

(i) $(\forall x)[P(x) \land Q(x)]$ hypothesis.

(ii) $P(x) \land Q(x)$ (i), ui.

(iii) $P(x)$ (ii), Simplification.

(iv) $(\forall x)P(x)$ (iii), ug.

(v) $Q(x)$ (ii), Simplification.

(vi) $(\forall x)Q(x)$ (v), ug.

(vii) $(\forall x)P(x) \land (\forall x)Q(x)$ (iv), (vi), Conjunction.

Note that neither restriction has been violated in the ug steps.

Exercise

Exercise

Example

Exercise

Example

Show that the following expression is valid:

$$[(\forall y)[P(x) \rightarrow Q(x, y)]] \rightarrow [P(x) \rightarrow (\forall y)Q(x, y)].$$

Exercise

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Proof

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Show that the following expression is valid:

$$[(\forall y)[P(x) \rightarrow Q(x,y)]] \rightarrow [P(x) \rightarrow (\forall y)Q(x,y)].$$

Proof

Using the Deduction Method, rewrite the argument as:

$$[(\forall y)[P(x) \rightarrow Q(x,y)] \land P(x)] \rightarrow (\forall y)Q(x,y)$$

Exercise

Example

Show that the following expression is valid:

$$[(\forall y)[P(x) \rightarrow Q(x,y)]] \rightarrow [P(x) \rightarrow (\forall y)Q(x,y)].$$

Proof

Using the Deduction Method, rewrite the argument as:

$$[(\forall y)[P(x) \rightarrow Q(x,y)] \land P(x)] \rightarrow (\forall y)Q(x,y)$$

Consider the following proof sequence:

Exercise

Example

Show that the following expression is valid:

$$[(\forall y)[P(x) \rightarrow Q(x,y)]] \rightarrow [P(x) \rightarrow (\forall y)Q(x,y)].$$

Proof

Using the Deduction Method, rewrite the argument as:

$$[(\forall y)[P(x) \rightarrow Q(x,y)] \wedge P(x)] \rightarrow (\forall y)Q(x,y)$$

Consider the following proof sequence:

(i) $(\forall y)[P(x) \rightarrow Q(x,y)]$ hypothesis.

Exercise

Example

Show that the following expression is valid:

$$[(\forall y)[P(x) \rightarrow Q(x,y)]] \rightarrow [P(x) \rightarrow (\forall y)Q(x,y)].$$

Proof

Using the Deduction Method, rewrite the argument as:

$$[(\forall y)[P(x) \rightarrow Q(x,y)] \land P(x)] \rightarrow (\forall y)Q(x,y)$$

Consider the following proof sequence:

(i) $(\forall y)[P(x) \rightarrow Q(x,y)]$ hypothesis.

(ii) $P(x) \rightarrow Q(x,y)$ (i), ui.

Exercise

Example

Show that the following expression is valid:

$$[(\forall y)[P(x) \rightarrow Q(x,y)]] \rightarrow [P(x) \rightarrow (\forall y)Q(x,y)].$$

Proof

Using the Deduction Method, rewrite the argument as:

$$[(\forall y)[P(x) \rightarrow Q(x,y)] \land P(x)] \rightarrow (\forall y)Q(x,y)$$

Consider the following proof sequence:

(i) $(\forall y)[P(x) \rightarrow Q(x,y)]$ hypothesis.

(ii) $P(x) \rightarrow Q(x,y)$ (i), ui.

(iii) $P(x)$ hypothesis.

Exercise

Example

Show that the following expression is valid:

$$[(\forall y)[P(x) \rightarrow Q(x,y)]] \rightarrow [P(x) \rightarrow (\forall y)Q(x,y)].$$

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Using the Deduction Method, rewrite the argument as:

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Consider the following proof sequence:

(i) $(\forall y)[P(x) \rightarrow Q(x,y)]$ hypothesis.
(ii) $P(x) \rightarrow Q(x,y)$ (i), ui.
(iii) $P(x)$ hypothesis.
(iv) $Q(x,y)$ (ii), (iii), Modus Ponens.

Exercise

Example

Show that the following expression is valid:

$$[(\forall y)[P(x) \rightarrow Q(x,y)]] \rightarrow [P(x) \rightarrow (\forall y)Q(x,y)].$$

Proof

Using the Deduction Method, rewrite the argument as:

$$[(\forall y)[P(x) \rightarrow Q(x,y)] \wedge P(x)] \rightarrow (\forall y)Q(x,y)$$

Consider the following proof sequence:

(i) $(\forall y)[P(x) \rightarrow Q(x,y)]$ hypothesis.
(ii) $P(x) \rightarrow Q(x,y)$ (i), ui.
(iii) $P(x)$ hypothesis.
(iv) $Q(x,y)$ (ii), (iii), Modus Ponens.
(v) $(\forall y)Q(x,y)$ (iv), ug.

A verbal argument

A verbal argument

Example

A verbal argument

Example

Every microcomputer has a serial interface port.

A verbal argument

Example

Every microcomputer has a serial interface port.

Some microcomputers have a parallel port.

A verbal argument

Example

Every microcomputer has a serial interface port.

Some microcomputers have a parallel port.

Therefore, some microcomputers have both a serial and a parallel port.

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Symbolically,

A verbal argument

Example

Every microcomputer has a serial interface port.

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Symbolically,

$$[(\forall x)[M(x) \rightarrow S(x)]$$

A verbal argument

Example

Every microcomputer has a serial interface port.

Some microcomputers have a parallel port.

Therefore, some microcomputers have both a serial and a parallel port.

Symbolically,

$$[(\forall x)[M(x) \rightarrow S(x)] \land (\exists x)[M(x) \land P(x)]]$$

A verbal argument

Example

Every microcomputer has a serial interface port.

Some microcomputers have a parallel port.

Therefore, some microcomputers have both a serial and a parallel port.

Symbolically,

$$[(\forall x)[M(x) \rightarrow S(x)] \land (\exists x)[M(x) \land P(x)]] \rightarrow (\exists x)[M(x) \land S(x) \land P(x)].$$

Proof of argument

Proof of argument

Proof.

Proof of argument

Proof.

Consider the following proof sequence:

Proof of argument

Proof.

Consider the following proof sequence:

(i) $(\exists x)[M(x) \wedge P(x)]$ hypothesis.

Proof of argument

Proof.

Consider the following proof sequence:

(i) $(\exists x)[M(x) \wedge P(x)]$ hypothesis.

(ii) $M(a) \wedge P(a)$ (ii), ei.

Proof of argument

Proof.

Consider the following proof sequence:

(i) $(\exists x)[M(x) \land P(x)]$ hypothesis.

(ii) $M(a) \land P(a)$ (ii), ei.

(iii) $(\forall x)[M(x) \rightarrow S(x)]$ hypothesis.

Proof of argument

Proof.

Consider the following proof sequence:

(i) $(\exists x)[M(x) \land P(x)]$ hypothesis.

(ii) $M(a) \land P(a)$ (ii), ei.

(iii) $(\forall x)[M(x) \to S(x)]$ hypothesis.

(iv) $M(a) \to S(a)$ (iii), ui.

Proof of argument

Proof.

Consider the following proof sequence:

(i) $(\exists x)[M(x) \land P(x)]$ hypothesis.

(ii) $M(a) \land P(a)$ (ii), ei.

(iii) $(\forall x)[M(x) \rightarrow S(x)]$ hypothesis.

(iv) $M(a) \rightarrow S(a)$ (iii), ui.

(v) $M(a)$ (iii), Simplification.

Proof of argument

Proof.

Consider the following proof sequence:

(i) $(\exists x)[M(x) \land P(x)]$ hypothesis.
(ii) $M(a) \land P(a)$ (ii), ei.
(iii) $(\forall x)[M(x) \to S(x)]$ hypothesis.
(iv) $M(a) \to S(a)$ (iii), ui.
(v) $M(a)$ (iii), Simplification.
(vi) $P(a)$ (iii), Simplification.

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Consider the following proof sequence:

(i) $(\exists x)[M(x) \land P(x)]$ hypothesis.
(ii) $M(a) \land P(a)$ (ii), ei.
(iii) $(\forall x)[M(x) \rightarrow S(x)]$ hypothesis.
(iv) $M(a) \rightarrow S(a)$ (iii), ui.
(v) $M(a)$ (iii), Simplification.
(vi) $P(a)$ (iii), Simplification.
(vii) $S(a)$ (iv), (v), Modus Ponens.

Proof of argument

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Consider the following proof sequence:

(i) $(\exists x)[M(x) \land P(x)]$ hypothesis.
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(iv) $M(a) \to S(a)$ (iii), ui.
(v) $M(a)$ (iii), Simplification.
(vi) $P(a)$ (iii), Simplification.
(vii) $S(a)$ (iv), (v), Modus Ponens.
(viii) $M(a) \land P(a) \land S(a)$ (iii), (vi), (vii), Conjunction.

Proof of argument

Proof.

Consider the following proof sequence:

(i) $(\exists x)[M(x) \wedge P(x)]$ hypothesis.

(ii) $M(a) \wedge P(a)$ (ii), ei.

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(iv) $M(a) \rightarrow S(a)$ (iii), ui.

(v) $M(a)$ (iii), Simplification.

(vi) $P(a)$ (iii), Simplification.

(vii) $S(a)$ (iv), (v), Modus Ponens.

(viii) $M(a) \wedge P(a) \wedge S(a)$ (iii), (vi), (vii), Conjunction.

(ix) $M(a) \wedge S(a) \wedge P(a)$ (viii), commutativity.

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Proof.

Consider the following proof sequence:

(i) $(\exists x)[M(x) \wedge P(x)]$ hypothesis.

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(v) $M(a)$ (iii), Simplification.

(vi) $P(a)$ (iii), Simplification.

(vii) $S(a)$ (iv), (v), Modus Ponens.

(viii) $M(a) \wedge P(a) \wedge S(a)$ (iii), (vi), (vii), Conjunction.

(ix) $M(a) \wedge S(a) \wedge P(a)$ (viii), commutativity.

(x) $(\exists x)[M(x) \wedge S(x) \wedge P(x)]$ (ix), eg.

□

Laws of Negation

Negation

Laws of Negation

Negation

(i) $[(\exists x)A(x)]' \Leftrightarrow (\forall x)[A(x)]'$.

Laws of Negation

Negation

(i) $[(\exists x)A(x)]' \Leftrightarrow (\forall x)[A(x)]'$.

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Laws of Negation

Negation

(i) $[(\exists x)A(x)]' \Leftrightarrow (\forall x)[A(x)]'$.

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Note

The proofs of the negation rules are tedious and I do not expect you to know them.

Laws of Negation

Negation

(i) $[(\exists x)A(x)]' \Leftrightarrow (\forall x)[A(x)]'$.

(ii) $[(\forall x)A(x)]' \Leftrightarrow (\exists x)[A(x)]'$.

Note

The proofs of the negation rules are tedious and I do not expect you to know them. Feel free to use the above two rules as and when you need them, without proof.

Example of negation

Example of negation

Example

Example of negation

Example

What is the logical negation of the statement,

Example of negation

Example

What is the logical negation of the statement, "Everybody loves somebody sometime."

Example of negation

Example

What is the logical negation of the statement, “Everybody loves somebody sometime.”

Solution:

Example of negation

Example

What is the logical negation of the statement, "Everybody loves somebody sometime."

Solution: Let the predicate $L(x, y, t)$ represent the property that x loves y at time t.

Example of negation

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What is the logical negation of the statement, "Everybody loves somebody sometime."

Solution: Let the predicate $L(x, y, t)$ represent the property that x loves y at time t.

The given English statement can therefore be represented as:

Example of negation

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What is the logical negation of the statement, “Everybody loves somebody sometime.”

Solution: Let the predicate $L(x, y, t)$ represent the property that x loves y at time t.

The given English statement can therefore be represented as:
$(\forall x)$

Example of negation

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What is the logical negation of the statement, "Everybody loves somebody sometime."

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What is the logical negation of the statement, "Everybody loves somebody sometime."

Solution: Let the predicate $L(x, y, t)$ represent the property that x loves y at time t.

The given English statement can therefore be represented as:
$(\forall x)(\exists y)(\exists t)L(x, y, t)$.

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Example

What is the logical negation of the statement, “Everybody loves somebody sometime.”

Solution: Let the predicate $L(x, y, t)$ represent the property that x loves y at time t.

The given English statement can therefore be represented as:
$(\forall x)(\exists y)(\exists t)L(x, y, t)$.

Its negation is therefore,

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Its negation is therefore,
$(\exists x)$

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$(\forall x)(\exists y)(\exists t)L(x, y, t)$.

Its negation is therefore,
$(\exists x)(\forall y)$

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Its negation is therefore,
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What is the logical negation of the statement, "Everybody loves somebody sometime."

Solution: Let the predicate $L(x, y, t)$ represent the property that x loves y at time t.

The given English statement can therefore be represented as:
$(\forall x)(\exists y)(\exists t)L(x, y, t)$.

Its negation is therefore,
$(\exists x)(\forall y)(\forall t)L(x, y, t)'$.

Example of negation

Example

What is the logical negation of the statement, “Everybody loves somebody sometime.”

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Its negation is therefore,
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In English, the negation is:

Example of negation

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What is the logical negation of the statement, "Everybody loves somebody sometime."

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The given English statement can therefore be represented as:
$(\forall x)(\exists y)(\exists t)L(x, y, t)$.

Its negation is therefore,
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In English, the negation is: There is somebody who

Example of negation

Example

What is the logical negation of the statement, "Everybody loves somebody sometime."

Solution: Let the predicate $L(x, y, t)$ represent the property that x loves y at time t.

The given English statement can therefore be represented as:
$(\forall x)(\exists y)(\exists t)L(x, y, t)$.

Its negation is therefore,
$(\exists x)(\forall y)(\forall t)L(x, y, t)'$.

In English, the negation is: There is somebody who does not love everybody,

Example of negation

Example

What is the logical negation of the statement, “Everybody loves somebody sometime.”

Solution: Let the predicate $L(x, y, t)$ represent the property that x loves y at time t.

The given English statement can therefore be represented as:
$(\forall x)(\exists y)(\exists t)L(x, y, t)$.

Its negation is therefore,
$(\exists x)(\forall y)(\forall t)L(x, y, t)'$.

In English, the negation is: There is somebody who does not love everybody, all the time. □

One more example

One more example

Example

One more example

Example

Establish the validity of the following statement:

$$(\forall x)[P(x) \vee Q(x)] \rightarrow [(\exists x)P(x) \vee (\forall x)Q(x)]$$

Solution to Example

Solution to Example

Proof

Solution to Example

Proof

We first rewrite the argument as: $(\forall x)[P(x) \lor Q(x)] \rightarrow [[(\exists x)P(x)]' \rightarrow (\forall x)Q(x)]$.

Solution to Example

Proof

We first rewrite the argument as: $(\forall x)[P(x) \vee Q(x)] \rightarrow [[(\exists x)P(x)]' \rightarrow (\forall x)Q(x)]$.

Using the Deduction Method, the revised argument can be further revised to

Solution to Example

Proof

We first rewrite the argument as: $(\forall x)[P(x) \vee Q(x)] \rightarrow [[(\exists x)P(x)]' \rightarrow (\forall x)Q(x)]$.

Using the Deduction Method, the revised argument can be further revised to

$[(\forall x)[P(x) \vee Q(x)] \wedge [(\exists x)P(x)]'] \rightarrow (\forall x)Q(x)$.

Solution to Example

Proof

We first rewrite the argument as: $(\forall x)[P(x) \lor Q(x)] \rightarrow [[(\exists x)P(x)]' \rightarrow (\forall x)Q(x)]$.

Using the Deduction Method, the revised argument can be further revised to

$[(\forall x)[P(x) \lor Q(x)] \land [(\exists x)P(x)]'] \rightarrow (\forall x)Q(x)$.

Solution to Example

Proof

We first rewrite the argument as: $(\forall x)[P(x) \vee Q(x)] \to [[(\exists x)P(x)]' \to (\forall x)Q(x)]$.

Using the Deduction Method, the revised argument can be further revised to

$[(\forall x)[P(x) \vee Q(x)] \wedge [(\exists x)P(x)]'] \to (\forall x)Q(x)$.

Consider the following proof sequence:

Solution to Example

Proof

We first rewrite the argument as: $(\forall x)[P(x) \lor Q(x)] \rightarrow [[(\exists x)P(x)]' \rightarrow (\forall x)Q(x)]$.

Using the Deduction Method, the revised argument can be further revised to

$[(\forall x)[P(x) \lor Q(x)] \land [(\exists x)P(x)]'] \rightarrow (\forall x)Q(x)$.

Consider the following proof sequence:

(i) $(\forall x)[P(x) \lor Q(x)]$ hypothesis.

Solution to Example

Proof

We first rewrite the argument as: $(\forall x)[P(x) \lor Q(x)] \rightarrow [[(\exists x)P(x)]' \rightarrow (\forall x)Q(x)]$.

Using the Deduction Method, the revised argument can be further revised to

$[(\forall x)[P(x) \lor Q(x)] \land [(\exists x)P(x)]'] \rightarrow (\forall x)Q(x)$.

Consider the following proof sequence:

(i) $(\forall x)[P(x) \lor Q(x)]$ hypothesis.
(ii) $[(\exists x)P(x)]'$ hypothesis.

Solution to Example

Proof

We first rewrite the argument as: $(\forall x)[P(x) \lor Q(x)] \rightarrow [[(\exists x)P(x)]' \rightarrow (\forall x)Q(x)]$.

Using the Deduction Method, the revised argument can be further revised to

$[(\forall x)[P(x) \lor Q(x)] \land [(\exists x)P(x)]'] \rightarrow (\forall x)Q(x)$.

Consider the following proof sequence:

(i) $(\forall x)[P(x) \lor Q(x)]$ hypothesis.
(ii) $[(\exists x)P(x)]'$ hypothesis.
(iii) $(\forall x)[P(x)]'$ (ii), negation.

Solution to Example

Proof

We first rewrite the argument as: $(\forall x)[P(x) \vee Q(x)] \rightarrow [[(\exists x)P(x)]' \rightarrow (\forall x)Q(x)]$.

Using the Deduction Method, the revised argument can be further revised to

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Consider the following proof sequence:

(i) $(\forall x)[P(x) \vee Q(x)]$ hypothesis.
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(iii) $(\forall x)[P(x)]'$ (ii), negation.
(iv) $[P(x)]'$ (iii), ui.

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(iv) $[P(x)]'$ (iii), ui.
(v) $P(x) \vee Q(x)$ (i), ui.

Solution to Example

Proof

We first rewrite the argument as: $(\forall x)[P(x) \lor Q(x)] \rightarrow [[(\exists x)P(x)]' \rightarrow (\forall x)Q(x)]$.

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Consider the following proof sequence:

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(ii) $[(\exists x)P(x)]'$ hypothesis.

(iii) $(\forall x)[P(x)]'$ (ii), negation.

(iv) $[P(x)]'$ (iii), ui.

(v) $P(x) \lor Q(x)$ (i), ui.

(vi) $[P(x)]' \rightarrow Q(x)$ (v), implication.

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Proof

We first rewrite the argument as: $(\forall x)[P(x) \lor Q(x)] \rightarrow [[(\exists x)P(x)]' \rightarrow (\forall x)Q(x)]$.

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(ii) $[(\exists x)P(x)]'$ hypothesis.
(iii) $(\forall x)[P(x)]'$ (ii), negation.
(iv) $[P(x)]'$ (iii), ui.
(v) $P(x) \lor Q(x)$ (i), ui.
(vi) $[P(x)]' \rightarrow Q(x)$ (v), implication.
(vii) $Q(x)$ (iv), (vi), Modus Ponens.

Solution to Example

Proof

We first rewrite the argument as: $(\forall x)[P(x) \vee Q(x)] \rightarrow [[(\exists x)P(x)]' \rightarrow (\forall x)Q(x)]$.

Using the Deduction Method, the revised argument can be further revised to

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(ii) $[(\exists x)P(x)]'$ hypothesis.
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(iv) $[P(x)]'$ (iii), ui.
(v) $P(x) \vee Q(x)$ (i), ui.
(vi) $[P(x)]' \rightarrow Q(x)$ (v), implication.
(vii) $Q(x)$ (iv), (vi), Modus Ponens.
(viii) $(\forall x)Q(x)$ (vii), ug.