Modeling

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Outline

1. Models and Model Types

Linear Programming

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1. Models and Model Types
2. Fundamental Steps
3. Assumptions of the linear programming model

Linear Programming

Outline

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Need for models

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Main Points

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Linear Programming

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3. Models can be “good” or “bad”.

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Main Points

1. Most interesting decision problems occur in large and complex systems.
2. The “what-if” question cannot be answered by trial and error.
3. Models can be “good” or “bad”.
4. The solution to a model is different from a solution to the actual system.

Types of Models

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1. Scale Model

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1. Scale Model - Aerodynamics.

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4. Matrix

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5. Mathematical Model

Types of Models

Model Types

1. Scale Model - Aerodynamics.
2. Pictorial Model - Spatial relationships.
3. Flow Chart for Networks - Functional relationships.
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5. Mathematical Model - Mathematical relationships captured by functions and equations.

Guidelines for Model Building

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General Guidelines

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1. Primary purpose of model.

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2. Requirements identification and Model type.

Guidelines for Model Building

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1. Primary purpose of model.
2. Requirements identification and Model type.
3. Level of detail.

Linear Programming

Guidelines for Model Building

General Guidelines

1. Primary purpose of model.
2. Requirements identification and Model type.
3. Level of detail.
4. Definition of input, output and relationships.

Important Terms

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Definitions

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1. Variable.

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1. Variable.
2. Decision variable.

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1. Variable.
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3. Continuous variable.

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1. Variable.
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6. Nonlinear function.
7. Mathematical model.
8. Equation.
9. Inequality.
10. Objective.
11. Constraint.

Basic Steps

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Formulating a linear program

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1. Determine the decision (or control or structural) variables.

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Formulating a linear program

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3. Formulate the constraints.

General Form of a Linear Program

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The general form of a linear programming is:

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$$\text{Optimize } z = c_1 \cdot x_1 + c_2 \cdot x_2 + \cdots + c_n \cdot x_n$$

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$$a_{1,1} \cdot x_1 + \cdots a_{1,n} \cdot x_n \{\leq, =, \text{ or } \geq\} b_1$$

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$$\vdots$$
$$a_{m,1} \cdot x_1 + \cdots a_{m,n} \cdot x_n \{\leq, =, \text{ or } \geq\} b_m$$

$$x_1, \ldots, x_n \geq 0$$

Compact representation

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Compact form

This can be written more compactly as

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$$x_i \geq 0 \text{ for } i = 1, \ldots, n$$

Linear Programming

Assumptions

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2. Proportionality

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2. Proportionality - Variable x_{ij} contributes $c_{ij} \cdot x_{ij}$ to the cost and $a_{ij} \cdot x_{ij}$ to the i^{th} constraint.

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3. Additivity - Total cost is the *sum* of cost contributions of each variable. No interactions reduce or increase the level of the combined contributions
4. Divisibility - Variables are continuous and not discrete.

Forms of a linear program

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1. General Form (Already discussed).

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2. Canonical form:

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$$\max \mathbf{c} \cdot \mathbf{x}$$

$$\begin{aligned} \mathbf{A} \cdot \mathbf{x} &\leq \mathbf{b} \\ \mathbf{x} &\geq \mathbf{0} \end{aligned}$$

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Converting linear programs into standard form

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Objective function

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Variables

If a variable (say x_1) is unrestricted in sign, replace it with $x_1' - x_1''$, where both $x_1', x_1'' \geq 0$.

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1. If a constraint is in the $\leq$ form, use a slack variable.

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Constraints

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Variables

If a variable (say x_1) is unrestricted in sign, replace it with $x_1' - x_1''$, where both $x_1', x_1'' \geq 0$.

Constraints

1. If a constraint is in the $\leq$ form, use a slack variable.
2. If a constraint is in the $\geq$ form, use a surplus variable.

Both slack and surplus variables are inherently non-negative.

Equivalence of the feasibility and optimization versions

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Exercise

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Exercise on constraint conversion

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Convert the following Linear Program into Standard Form.

$$\text{Minimize } z = 2 \cdot x_1 - 3 \cdot x_2 + 5 \cdot x_3 + x_4$$

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Convert the following Linear Program into Standard Form.

$$\text{Minimize } z = 2 \cdot x_1 - 3 \cdot x_2 + 5 \cdot x_3 + x_4$$

subject to

$$\begin{aligned}
-x_1 + 3 \cdot x_2 - x_3 + 2 \cdot x_4 &\leq -12 \\
5 \cdot x_1 + x_2 + 4 \cdot x_3 - x_4 &\geq 10 \\
3 \cdot x_1 - 2 \cdot x_2 + x_3 - x_4 &= -8 \\
x_1, x_2, x_3, x_4 &\geq 0
\end{aligned}$$

Solution

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$$-x_1 + 3 \cdot x_2 - x_3 + 2 \cdot x_4 + s_1 = -12$$
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Adding the bounds on the slack and surplus variables

$$x_1, x_2, x_3, x_4, s_1, s_2 \geq 0$$

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Converting the constraints we get,

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Adding the bounds on the slack and surplus variables

$$x_1, x_2, x_3, x_4, s_1, s_2 \geq 0$$

Finally, converting the objective function

Solution

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Converting the constraints we get,

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Adding the bounds on the slack and surplus variables

$$x_1, x_2, x_3, x_4, s_1, s_2 \geq 0$$

Finally, converting the objective function

$$\text{Maximize } z = -2 \cdot x_1 + 3 \cdot x_2 - 5 \cdot x_3 - x_4 + 0s_1 + 0s_2$$

A Classification problem

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Example

A Classification problem

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1. Assume that you measure the heights and weights of n Danish men and m Danish women.

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The goal is to find a linear classifier

A Classification problem

Example

1. Assume that you measure the heights and weights of n Danish men and m Danish women.
2. This data is plotted as points on the $x-y$ plane.

The goal is to find a linear classifier (straight line) that separates the points representing the Danish men from the Danish women.

A product mix problem

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Example

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We have two gadgets to produce: α and β.

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We have two gadgets to produce: α and β.

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2. Each unit of α requires 4 hours of assembly and 1 hour of testing.
3. The return for a unit of β is \$30.

A product mix problem

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We have two gadgets to produce: α and β.

1. The return for a unit of α is \$20.
2. Each unit of α requires 4 hours of assembly and 1 hour of testing.
3. The return for a unit of β is \$30.
4. Each unit of β requires 3 hours of assembly and 2 hours of testing.

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5. We must produce at least 25 units of α.

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5. We must produce at least 25 units of α.
6. We have a total of 240 hours available for assembly and 140 hours for testing.

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How many units of α and β should be produced to maximize our return?

Modeling the product mix problem

Modeling the product mix problem

Decision Variables

Modeling the product mix problem

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Let x_1 denote the number of units of α and x_2 denote the number of units of β to be manufactured.

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Objective function

Modeling the product mix problem

Decision Variables

Let x_1 denote the number of units of α and x_2 denote the number of units of β to be manufactured.

Objective function

$\max 20 \cdot x_1 + 30 \cdot x_2.$

 Linear Programming

Modeling the product mix problem

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Constraints

Linear Programming

Modeling the product mix problem

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$$4 \cdot x_1 + 3 \cdot x_2 \quad \leq \quad 240$$

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$$\begin{aligned} 4 \cdot x_1 + 3 \cdot x_2 &\leq 240 \\ x_1 + 2 \cdot x_2 &\leq 140 \end{aligned}$$

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Linear Programming

Portfolio optimization

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Linear Programming

Portfolio optimization

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2. We will *not* re-invest the interest at the end of the year.
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Portfolio optimization

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1. The annual return on each investment is 7%, 9%, and 14% respectively.
2. We will *not* re-invest the interest at the end of the year.
3. We do not want to invest less than $\$10,000$ in bonds.
4. The investment in stocks should not exceed the combined total investment in the other two strategies.

Portfolio optimization

Example

We want to invest $\$50,000$ among three strategies: savings certificates, municipal bonds, and stocks.

1. The annual return on each investment is 7%, 9%, and 14% respectively.
2. We will *not* re-invest the interest at the end of the year.
3. We do not want to invest less than $\$10,000$ in bonds.
4. The investment in stocks should not exceed the combined total investment in the other two strategies.
5. The savings certificate investment should be between $\$5,000$ and $\$15,000$.

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How should we invest the money in order to maximize our return?

Modeling the portfolio optimization problem

Modeling the portfolio optimization problem

Decision Variables

Modeling the portfolio optimization problem

Decision Variables

Let x_1, x_2 and x_3 denote the amounts to be invested in savings certificates, municipal bonds and stocks respectively.

Linear Programming

Modeling the portfolio optimization problem

Decision Variables

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Objective Function

Modeling the portfolio optimization problem

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Let x_1, x_2 and x_3 denote the amounts to be invested in savings certificates, municipal bonds and stocks respectively.

Objective Function

$\max 0.07 \cdot x_1 + 0.09 \cdot x_2 + 0.14 \cdot x_3$.

Modeling the portfolio optimization problem

Decision Variables

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Constraints

Modeling the portfolio optimization problem

Decision Variables

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Constraints

Linear Programming

Modeling the portfolio optimization problem

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Objective Function

$\max 0.07 \cdot x_1 + 0.09 \cdot x_2 + 0.14 \cdot x_3$.

Constraints

$$x_2 \geq 10,000$$

Modeling the portfolio optimization problem

Decision Variables

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Objective Function

$\max 0.07 \cdot x_1 + 0.09 \cdot x_2 + 0.14 \cdot x_3$.

Constraints

$$\begin{aligned} x_2 &\geq 10,000 \\ x_3 &\leq x_1 + x_2 \end{aligned}$$

Modeling the portfolio optimization problem

Decision Variables

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Objective Function

$\max 0.07 \cdot x_1 + 0.09 \cdot x_2 + 0.14 \cdot x_3$.

Constraints

$$\begin{array}{rcl} x_2 & \geq & 10,000 \\ x_3 & \leq & x_1 + x_2 \\ x_1 & \geq & 5000 \end{array}$$

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\begin{array}{rcl}
x_2 & \geq & 10,000 \\
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\end{array}
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$$
\begin{aligned}
x_2 &\geq 10,000 \\
x_3 &\leq x_1 + x_2 \\
x_1 &\geq 5000 \\
x_1 &\leq 15,000 \\
x_1 + x_2 + x_3 &\leq 50,000
\end{aligned}
$$

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$\max 0.07 \cdot x_1 + 0.09 \cdot x_2 + 0.14 \cdot x_3.$

Constraints

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\begin{aligned}
x_2 &\geq 10,000 \\
x_3 &\leq x_1 + x_2 \\
x_1 &\geq 5000 \\
x_1 &\leq 15,000 \\
x_1 + x_2 + x_3 &\leq 50,000 \\
x_1, x_2, x_3 &\geq 0
\end{aligned}
$$

Farmland Use

Farmland Use

Example

Farmland Use

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We own 500 acres of land, in which we grow corn, wheat, soybeans and oats.

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Farmland Use

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3. We require at least $10,000$ bushels of corn product due to a contract with a local dairy farm.

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4. The total wheat acreage should not be less than that used for soybeans and oats.

Linear Programming

Farmland Use

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4. The total wheat acreage should not be less than that used for soybeans and oats.
5. The selling price per bushel of corn is \$0.36; of wheat, \$0.90; of soybeans, \$0.82; of oats, \$0.98.

Linear Programming

Farmland Use

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How many acres of each product should be grown to maximize our profit?

Linear Programming

Modeling the Farmland Use problem

Modeling the Farmland Use problem

Decision Variables

Modeling the Farmland Use problem

Decision Variables

Let x_1, x_2, x_3 and x_4 denote the acreage of corn, what, soybeans and oats respectively.

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Objective Function

Modeling the Farmland Use problem

Decision Variables

Let x_1, x_2, x_3 and x_4 denote the acreage of corn, what, soybeans and oats respectively.

Objective Function

$\max(0.36) \cdot 110 \cdot x_1 + (0.9) \cdot 35 \cdot x_2 + (0.82) \cdot 32 \cdot x_3 + (0.98) \cdot 55 \cdot x_4.$

Modeling the Farmland Use problem

Decision Variables

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Objective Function

$\max(0.36) \cdot 110 \cdot x_1 + (0.9) \cdot 35 \cdot x_2 + (0.82) \cdot 32 \cdot x_3 + (0.98) \cdot 55 \cdot x_4.$

Constraints

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Constraints

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$\max(0.36) \cdot 110 \cdot x_1 + (0.9) \cdot 35 \cdot x_2 + (0.82) \cdot 32 \cdot x_3 + (0.98) \cdot 55 \cdot x_4.$

Constraints

$$x_1 + x_2 + x_3 + x_4 \quad \leq \quad 500$$

Modeling the Farmland Use problem

Decision Variables

Let x_1, x_2, x_3 and x_4 denote the acreage of corn, what, soybeans and oats respectively.

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$\max(0.36) \cdot 110 \cdot x_1 + (0.9) \cdot 35 \cdot x_2 + (0.82) \cdot 32 \cdot x_3 + (0.98) \cdot 55 \cdot x_4.$

Constraints

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &\leq 500 \\ x_3 &\leq 120 \end{aligned}$$

Linear Programming

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Constraints

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &\leq 500 \\ x_3 &\leq 120 \\ 110 \cdot x_1 &\geq 10,000 \end{aligned}$$

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Constraints

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &\leq 500 \\ x_3 &\leq 120 \\ 110 \cdot x_1 &\geq 10,000 \\ x_2 &\geq x_3 + x_4 \end{aligned}$$

Linear Programming

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Constraints

$$
\begin{aligned}
x_1 + x_2 + x_3 + x_4 &\leq 500 \\
x_3 &\leq 120 \\
110 \cdot x_1 &\geq 10,000 \\
x_2 &\geq x_3 + x_4 \\
x_1, x_2, x_3, x_4 &\geq 0
\end{aligned}
$$

Transportation

Transportation

Example

Transportation

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We have three warehouses and four clients.

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1. Warehouses 1, 2, and 3 have $6,000$, $9,000$, and $4,000$ units available respectively.

Transportation

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1. Warehouses 1, 2, and 3 have $6,000$, $9,000$, and $4,000$ units available respectively.
2. Clients 1, 2, 3, and 4 want $3,900$, $5,200$, $2,700$, and $6,400$ units respectively.

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3. The cost to ship a unit from a given warehouse to a given client varies according to the following table:

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	Client			
Warehouse	1	2	3	4
1	7	3	8	4
2	8	5	6	3
3	4	6	9	6

Transportation

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4. Items should be shipped from warehouses to clients, so all client demands are met.

Transportation

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3	4	6	9	6

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How can we perform the shipping while minimizing our shipping cost?

Modeling the Transportation problem

Modeling the Transportation problem

Decision Variables

Modeling the Transportation problem

Decision Variables

Let $x_{i,j}$ denote the number of units to be shipped from warehouse i to client j, where $i \in \{1, 2, 3\}$ and $j \in \{1, 2, 3, 4\}$.

Linear Programming

Modeling the Transportation problem

Decision Variables

Let $x_{i,j}$ denote the number of units to be shipped from warehouse i to client j, where $i \in \{1, 2, 3\}$ and $j \in \{1, 2, 3, 4\}$.

Objective Function

Modeling the Transportation problem

Decision Variables

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Objective Function

$$\begin{aligned} \min \quad & 7 \cdot x_{1,1} + 3 \cdot x_{1,2} + 8 \cdot x_{1,3} + 4 \cdot x_{1,4} \\ & 9 \cdot x_{2,1} + 5 \cdot x_{2,2} + 6 \cdot x_{2,3} + 3 \cdot x_{2,4} \\ & 4 \cdot x_{3,1} + 6 \cdot x_{3,2} + 9 \cdot x_{3,3} + 6 \cdot x_{3,4} \end{aligned}$$

Modeling (contd.)

Modeling (contd.)

Constraints

Modeling (contd.)

Constraints

The supply constraints:

Modeling (contd.)

Constraints

The supply constraints:

$$\sum_{j=1}^{4} x_{1,j} \leq 6000$$

Modeling (contd.)

Constraints

The supply constraints:

$$\begin{aligned} \sum_{j=1}^{4} x_{1,j} &\leq 6000 \\ \sum_{j=1}^{4} x_{2,j} &\leq 9000 \end{aligned}$$

Modeling (contd.)

Constraints

The supply constraints:

$$\begin{aligned}
\sum_{j=1}^{4} x_{1,j} &\leq 6000 \\
\sum_{j=1}^{4} x_{2,j} &\leq 9000 \\
\sum_{j=1}^{4} x_{3,j} &\leq 4000
\end{aligned}$$

Modeling (contd.)

Constraints

The supply constraints:

$$\begin{aligned}
\sum_{j=1}^{4} x_{1,j} &\leq 6000 \\
\sum_{j=1}^{4} x_{2,j} &\leq 9000 \\
\sum_{j=1}^{4} x_{3,j} &\leq 4000
\end{aligned}$$

The demand constraints:

Modeling (contd.)

Constraints

The supply constraints:

$$\sum_{j=1}^{4} x_{1,j} \leq 6000$$

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$$\sum_{j=1}^{4} x_{3,j} \leq 4000$$

The demand constraints:

$$\sum_{i=1}^{3} x_{i,1} = 3900$$

Modeling (contd.)

Constraints

The supply constraints:

$$\sum_{j=1}^{4} x_{1,j} \leq 6000$$

$$\sum_{j=1}^{4} x_{2,j} \leq 9000$$

$$\sum_{j=1}^{4} x_{3,j} \leq 4000$$

The demand constraints:

$$\sum_{i=1}^{3} x_{i,1} = 3900$$

$$\sum_{i=1}^{3} x_{i,2} = 5200$$

Modeling (contd.)

Constraints

The supply constraints:

$$\sum_{j=1}^{4} x_{1,j} \leq 6000$$

$$\sum_{j=1}^{4} x_{2,j} \leq 9000$$

$$\sum_{j=1}^{4} x_{3,j} \leq 4000$$

The demand constraints:

$$\sum_{i=1}^{3} x_{i,1} = 3900$$

$$\sum_{i=1}^{3} x_{i,2} = 5200$$

$$\sum_{i=1}^{3} x_{i,3} = 2700$$

Modeling (contd.)

Constraints

The supply constraints:

$$\sum_{j=1}^{4} x_{1,j} \leq 6000$$

$$\sum_{j=1}^{4} x_{2,j} \leq 9000$$

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The demand constraints:

$$\sum_{i=1}^{3} x_{i,1} = 3900$$

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$$\sum_{i=1}^{3} x_{i,3} = 2700$$

$$\sum_{i=1}^{3} x_{i,4} = 6400$$

Modeling (contd.)

Constraints

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The demand constraints:

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$$\sum_{i=1}^{3} x_{i,4} = 6400$$

Non-negativity constraints:
$x_{ij} \geq 0,\ i = 1, 2, 3,\ j = 1, 2, 3, 4.$

Short-term financing

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The problem

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The problem

1. Companies routinely face the problem of short-term commitments.

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3. Consider the following table:

Month	Jan	Feb	March	April	May	June
Net Cash flow	$-150\,K$	$-100\,K$	$200\,K$	$-200\,K$	$50\,K$	$300\,K$

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4. The company has a credit line of \$100 K at an interest rate of 1% per month.

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5. In any of the first three months, it can issue 90-day commercial paper.

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5. In any of the first three months, it can issue 90-day commercial paper.
6. Excess funds can be reinvested at an interest rate of 0.3% per month.

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5. In any of the first three months, it can issue 90-day commercial paper.
6. Excess funds can be reinvested at an interest rate of 0.3% per month.
7. Any paper issued in January through March requires a 2% interest rate payment three months later.

Modeling

Modeling

Decision Variables

Modeling

Decision Variables

Let

Modeling

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Let

1. x_i denote the amount drawn from the credit line in month i,

Modeling

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2. y_i denote the amount of commercial paper issued in month i,

Modeling

Decision Variables

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2. y_i denote the amount of commercial paper issued in month i,
3. z_i denote the excess funds in month i, and

Modeling

Decision Variables

Let

1. x_i denote the amount drawn from the credit line in month i,
2. y_i denote the amount of commercial paper issued in month i,
3. z_i denote the excess funds in month i, and
4. v denote the company's wealth after June.

Modeling

Decision Variables

Let

1. x_i denote the amount drawn from the credit line in month i,
2. y_i denote the amount of commercial paper issued in month i,
3. z_i denote the excess funds in month i, and
4. v denote the company's wealth after June.

Objective Function

Modeling

Decision Variables

Let

1. x_i denote the amount drawn from the credit line in month i,
2. y_i denote the amount of commercial paper issued in month i,
3. z_i denote the excess funds in month i, and
4. v denote the company's wealth after June.

Objective Function

$$\max v$$

Modeling (contd.)

Modeling (contd.)

Constraints

Modeling (contd.)

Constraints

$$x_1 + y_1 - z_1 = 150$$

Modeling (contd.)

Constraints

$$\begin{aligned} x_1 + y_1 - z_1 &= 150 \\ x_2 + y_2 - 1.01 \cdot x_1 + 1.003 \cdot z_1 - z_2 &= 100 \end{aligned}$$

Modeling (contd.)

Constraints

$$\begin{aligned}
x_1 + y_1 - z_1 &= 150 \\
x_2 + y_2 - 1.01 \cdot x_1 + 1.003 \cdot z_1 - z_2 &= 100 \\
x_3 + y_3 - 1.01 \cdot x_2 + 1.003 \cdot z_2 - z_3 &= -200
\end{aligned}$$

Linear Programming

Modeling (contd.)

Constraints

$$
\begin{aligned}
x_1 + y_1 - z_1 &= 150 \\
x_2 + y_2 - 1.01 \cdot x_1 + 1.003 \cdot z_1 - z_2 &= 100 \\
x_3 + y_3 - 1.01 \cdot x_2 + 1.003 \cdot z_2 - z_3 &= -200 \\
x_4 - 1.02 \cdot y_1 - 1.01 \cdot x_3 + 1.003 \cdot z_3 - z_4 &= 200
\end{aligned}
$$

Modeling (contd.)

Constraints

$$
\begin{aligned}
x_1 + y_1 - z_1 &= 150 \\
x_2 + y_2 - 1.01 \cdot x_1 + 1.003 \cdot z_1 - z_2 &= 100 \\
x_3 + y_3 - 1.01 \cdot x_2 + 1.003 \cdot z_2 - z_3 &= -200 \\
x_4 - 1.02 \cdot y_1 - 1.01 \cdot x_3 + 1.003 \cdot z_3 - z_4 &= 200 \\
x_5 - 1.02 \cdot y_2 - 1.01 \cdot x_4 + 1.003 \cdot z_4 - z_5 &= -50
\end{aligned}
$$

Modeling (contd.)

Constraints

$$\begin{aligned}
x_1 + y_1 - z_1 &= 150 \\
x_2 + y_2 - 1.01 \cdot x_1 + 1.003 \cdot z_1 - z_2 &= 100 \\
x_3 + y_3 - 1.01 \cdot x_2 + 1.003 \cdot z_2 - z_3 &= -200 \\
x_4 - 1.02 \cdot y_1 - 1.01 \cdot x_3 + 1.003 \cdot z_3 - z_4 &= 200 \\
x_5 - 1.02 \cdot y_2 - 1.01 \cdot x_4 + 1.003 \cdot z_4 - z_5 &= -50 \\
-1.02 \cdot y_3 - 1.01 \cdot x_5 + 1.003 \cdot z_5 - v &= -300
\end{aligned}$$

Linear Programming

Modeling (contd.)

Constraints

$$
\begin{array}{rcl}
x_1 + y_1 - z_1 & = & 150 \\
x_2 + y_2 - 1.01 \cdot x_1 + 1.003 \cdot z_1 - z_2 & = & 100 \\
x_3 + y_3 - 1.01 \cdot x_2 + 1.003 \cdot z_2 - z_3 & = & -200 \\
x_4 - 1.02 \cdot y_1 - 1.01 \cdot x_3 + 1.003 \cdot z_3 - z_4 & = & 200 \\
x_5 - 1.02 \cdot y_2 - 1.01 \cdot x_4 + 1.003 \cdot z_4 - z_5 & = & -50 \\
-1.02 \cdot y_3 - 1.01 \cdot x_5 + 1.003 \cdot z_5 - v & = & -300 \\
x_i & \leq & 100, \ i = 1, 2, 3, 4, 5
\end{array}
$$

Modeling (contd.)

Constraints

$$
\begin{aligned}
x_1 + y_1 - z_1 &= 150 \\
x_2 + y_2 - 1.01 \cdot x_1 + 1.003 \cdot z_1 - z_2 &= 100 \\
x_3 + y_3 - 1.01 \cdot x_2 + 1.003 \cdot z_2 - z_3 &= -200 \\
x_4 - 1.02 \cdot y_1 - 1.01 \cdot x_3 + 1.003 \cdot z_3 - z_4 &= 200 \\
x_5 - 1.02 \cdot y_2 - 1.01 \cdot x_4 + 1.003 \cdot z_4 - z_5 &= -50 \\
-1.02 \cdot y_3 - 1.01 \cdot x_5 + 1.003 \cdot z_5 - v &= -300 \\
x_i &\leq 100, \ i = 1, 2, 3, 4, 5 \\
x_i, y_i, z_i &\geq 0
\end{aligned}
$$

Final points

Final points

Choice of variables

Final points

Choice of variables

1. Why was the interest on x_1 not included in the equation for March?

Final points

Choice of variables

1. Why was the interest on x_1 not included in the equation for March?
2. Choice of variables is crucial.