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- Computationally very different from Eulerian paths
- Note that HAMILTONIAN is in **NP**
- We reduce 3-CNF SAT to HAMILTONIAN
  - For a fixed 3-CNF SAT formula, show that it can be transformed into a graph whose Hamiltonian path will give us the assignments for SAT.

Reduction Sketch

$$\phi = \mathcal{C}_1 \wedge \mathcal{C}_2 \wedge \ldots \wedge \mathcal{C}_m, \quad n \text{ variables}, \quad \mathcal{C}_i = (x_1^i \vee x_2^i \vee x_3^i)$$

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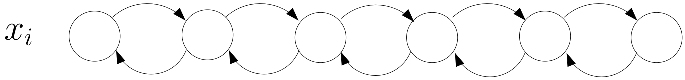

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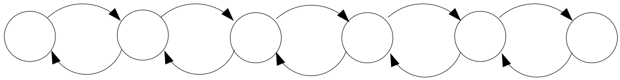
- Given ϕ, have to construct a graph G such that G has a Hamiltonian path iff ϕ is satisfiable.
- We first define the mapping of variables, and then the clauses.
- Each x_i will correspond to a path (chain) of $6m$ vertices
- If we are at the first (or end) vertex, only one path to follow

x_i



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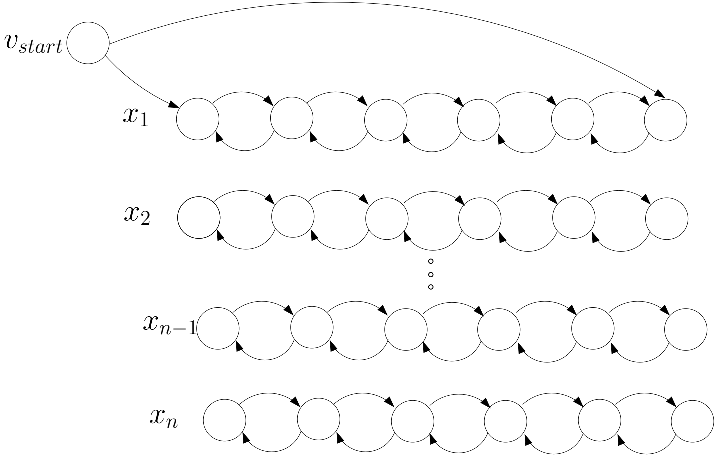

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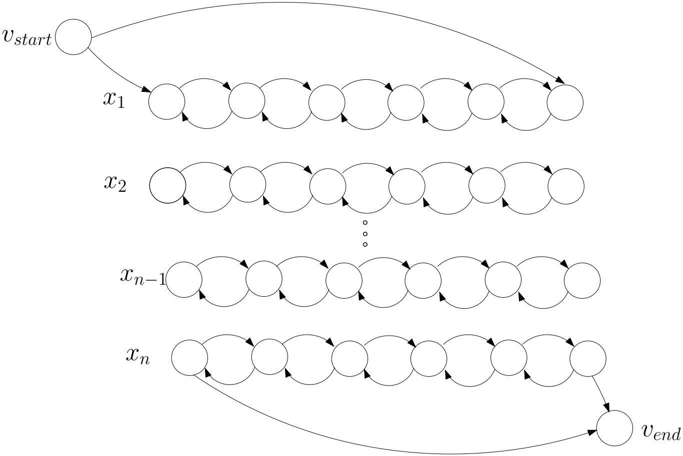

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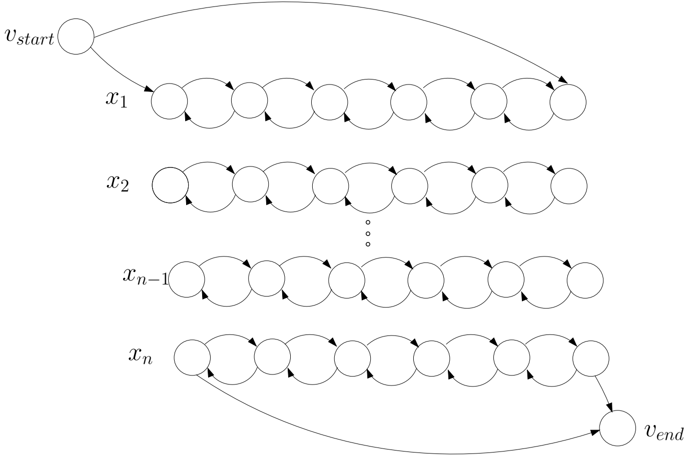

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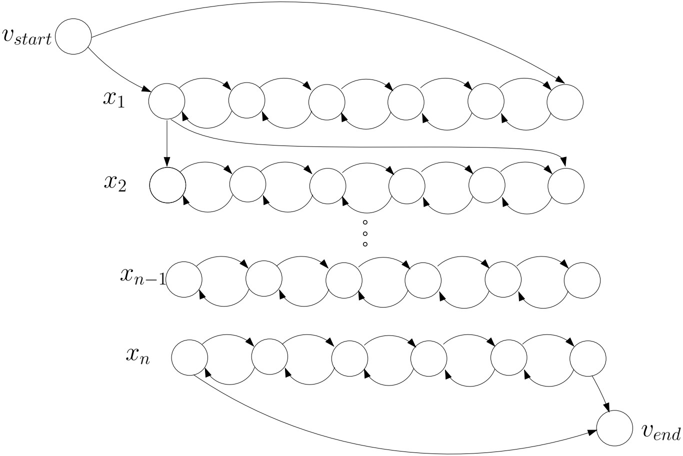

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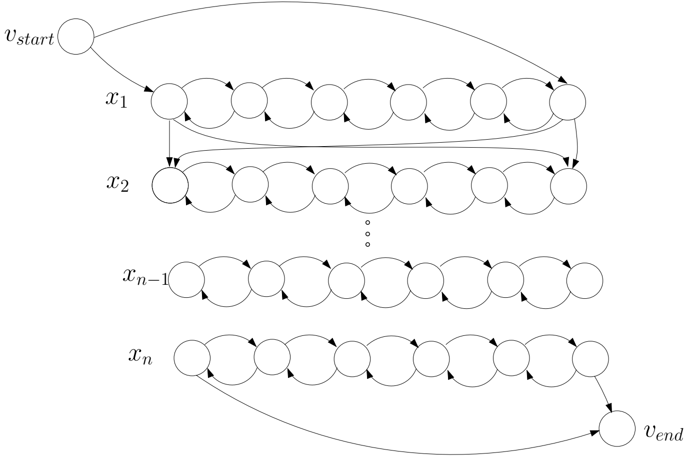

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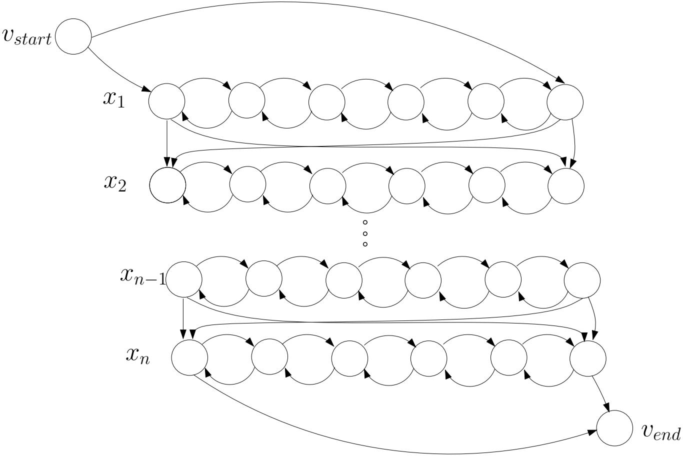
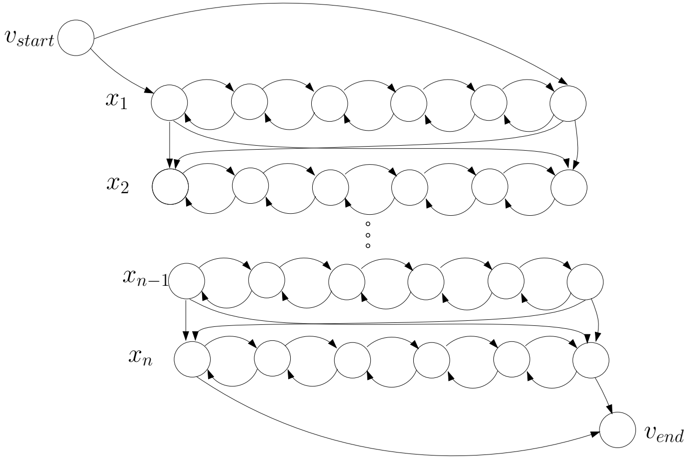
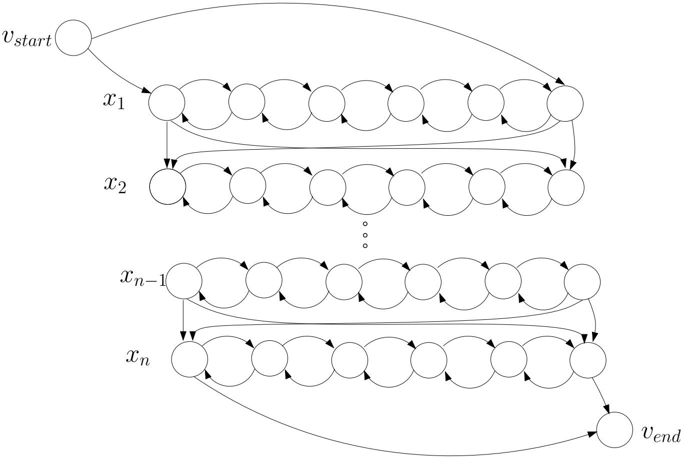


Construction Properties



Construction Properties

- Any Hamiltonian path has to start at v_{start}



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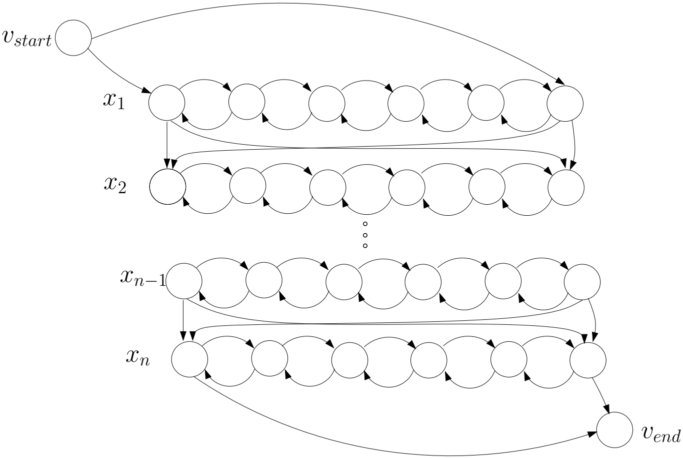
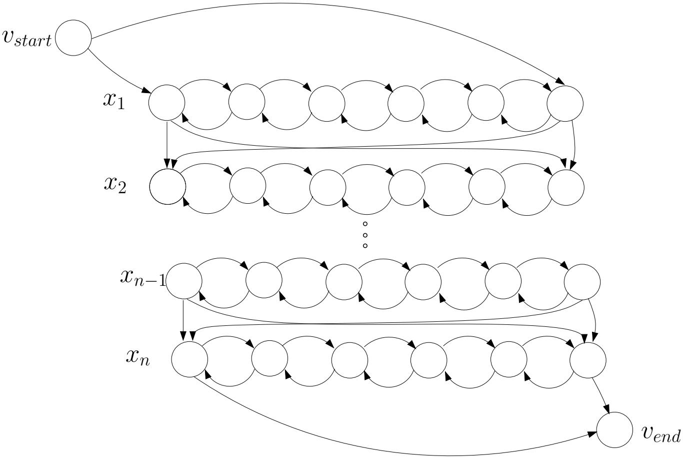


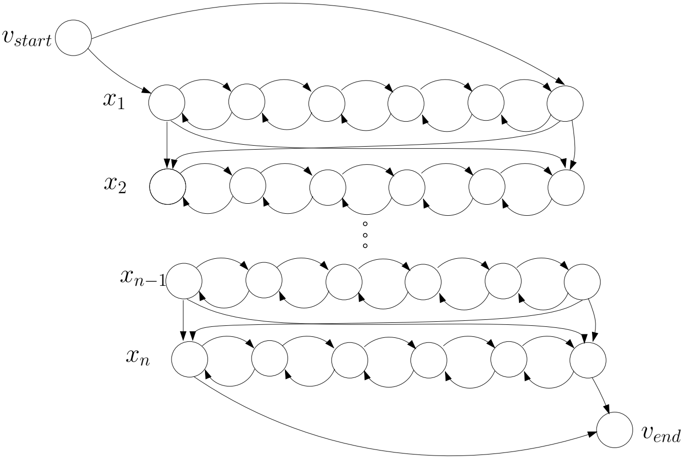
Construction Properties

- Any Hamiltonian path first traverses chain x_1, then x_2 etc.



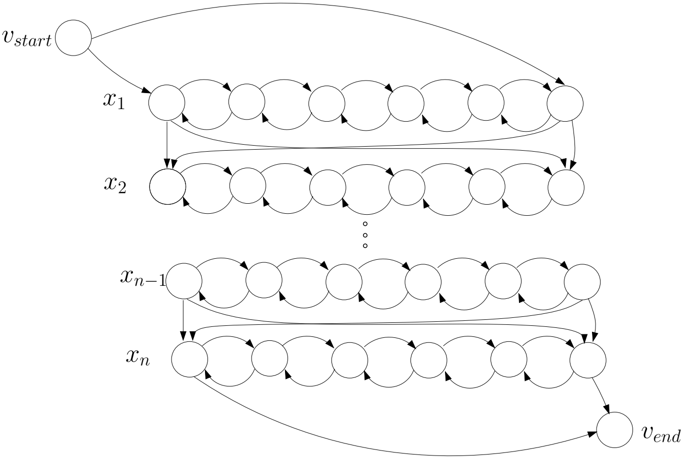
Construction Properties

- For each chain, only two ways of traversing it.
  - Left-to-right means $x_i = 1$, right-to-left means $x_i = 0$



Construction Properties

- Each assignment of variables corresponds to a unique Hamiltonian path.



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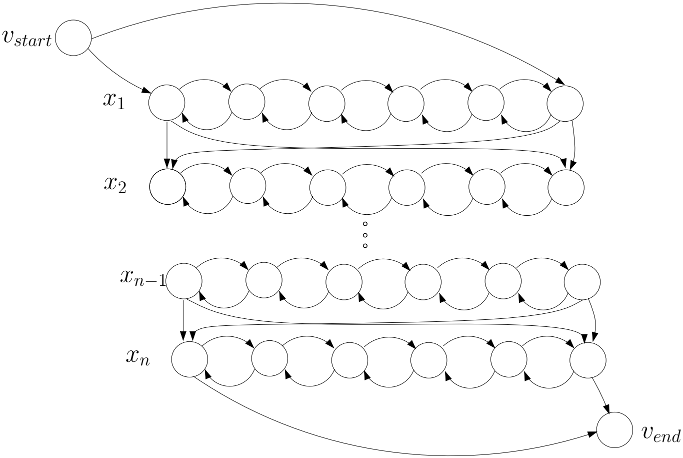
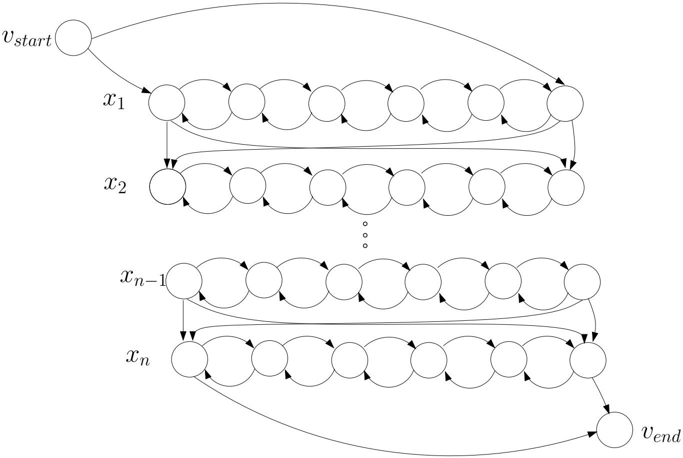


Construction Properties

- So far, no constraints – they will come from the clauses now.



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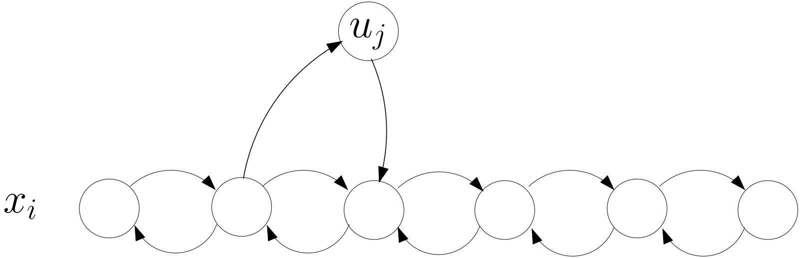

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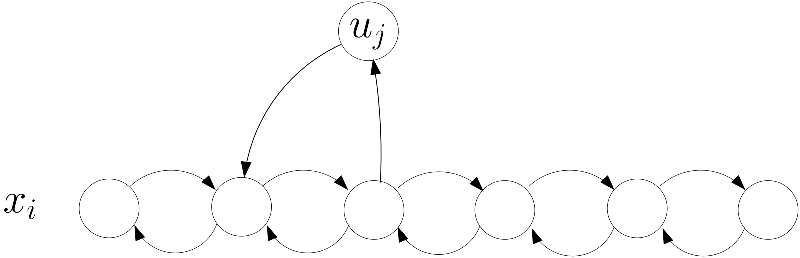


An Example

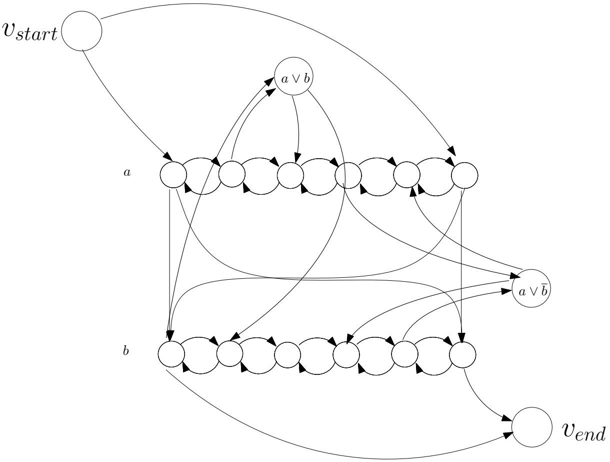
Construction of Hamiltonian path for

$$(a \vee b) \wedge (a \vee \overline{b})$$

Here:

- $\mathcal{C}_1 = (a \vee b)$
- $\mathcal{C}_2 = (a \vee \overline{b})$

The Graph Construction



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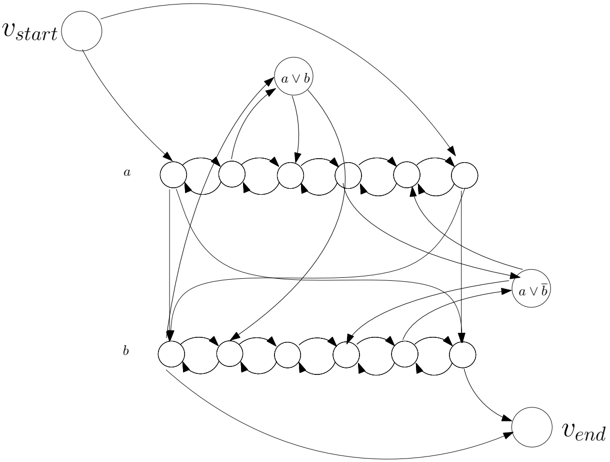
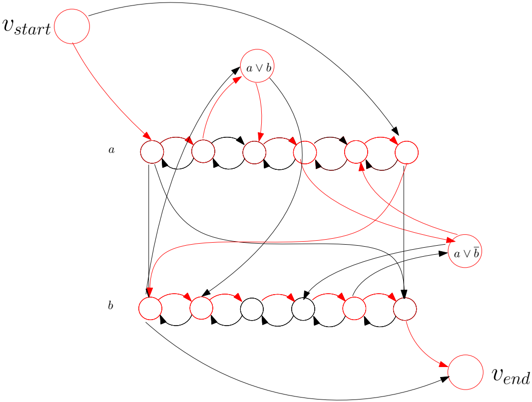


Claim

Hamiltonian path exists ONLY if you go from left to right in *a* and chose any one of the two directions for *b*.

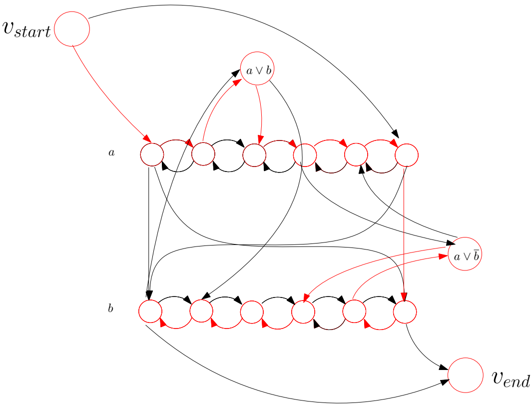
Path Corresponding to Assignment 1

$a = 1; b = 1$

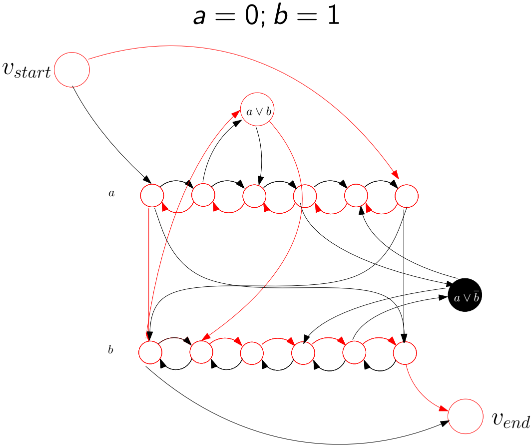


Path Corresponding to Assignment 2

$a = 1; b = 0$



Path Corresponding to Assignment 3

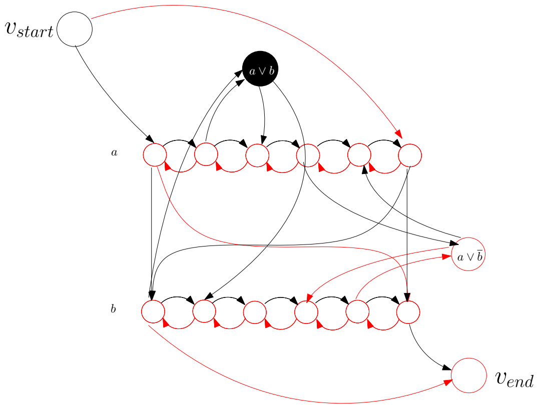


Error in finding HAMILTONIAN

No HAMILTONIAN PATH as $(a \vee \overline{b})$ is not accessible

Path Corresponding to Assignment 4

$a = 0; b = 0$



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- The above two *only* ways to visit u_j without getting stuck.