# How can I reduce Subset Sum to Partition?

Maybe this is quite simple but I have some trouble to get this reduction. I want to reduce Subset Sum to Partition but at this time I don't see the relation!

Is it possible to reduce this problem using a Levin Reduction ?

If you don't understand write for clarification!

complexity-theory np-complete reductions

edited Oct 16 '12 at 18:57

asked Oct 16 '12 at 18:04

dbonadiman
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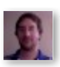
## 2 Answers

Let $(L, B)$ be an instance of subset sum, where $L$ is a list (multiset) of numbers, and $B$ is the target sum. Let $S = \sum L$. Let $L'$ be the list formed by adding $S + B, 2S - B$ to $L$.

(1) If there is a sublist $M \subseteq L$ summing to $B$, then $L'$ can be partitioned into two equal parts: $M \cup \{2S - B\}$ and $L \setminus M \cup \{S + B\}$. Indeed, the first part sums to $B + (2S - B) = 2S$, and the second to $(S - B) + (S + B) = 2S$.

(2) If $L'$ can be partitioned into two equal parts $P_1, P_2$, then there is a sublist of $L$ summing to $B$. Indeed, since $(S + B) + (2S - B) = 3S$ and each part sums to $2S$, the two elements belong to different parts. Without loss of generality, $2S - B \in P_1$. The rest of the elements in $P_1$ belong to $L$ and sum to $B$.

answered Oct 16 '12 at 21:49



Yuval Filmus
26.2k 2 21 59

But the standard subset-sum problem uses all integers, and partition problem uses just non-negative integers... – Harold Apr 19 at 17:40

SUBSET-SUM is NP-complete even with non-negative integers, for example the reduction from 3SAT ends up with non-negative integers. Also, there is probably a direct reduction from integer SUBSET-SUM to non-negative integer SUBSET-SUM. – Yuval Filmus Apr 19 at 20:26

Yes, I know, and this reduction is very easy. Just noting that it isn't the subset sum in it's "default" form. :) – Harold Apr 20 at 3:11

**Subset Sum:**

Input: {a1,a2,...,am} s.t M={1..m} and ai are non negative integer and S⊆{1..k} and Σai(i∈S) = t

**Partition:**

Input: {a1,a2,...,am} and S⊆ {1,· · ·,m} s.t Σai(i∈S) = Σaj(j∉S)

**Partition Np Proof:** if prover provides a partitions(P1,P2) for verifier, verifier can easily calculate the sum of P1 and P2 and check if the result is 0 in linear time.

**NP_Hard: SubsetSum ≤p PARTITION**

Let x be input of SubsetSum and x= 〈a1,a2,...,am,t〉 and Σai(i from 1 to m) = a

*Case1: 2t >= a:*

Let f(x)= 〈a1,a2,...,am,am+1〉 where am+1= 2t−a

We want to show that **x∈SubsetSum ⇔ f(x)∈PARTITION**

so there exist S⊆ {1,...,m} s.t T = {1..m} - S and Σai(i∈T) = a-t

and Let T' = {1...m,m+1} - S so Σaj(j∈T') = a-t+2t-a = t

which is exactly Σai(i∈S)= t and it shows f(x)∈PARTITION

now We also will show that **f(x)∈PARTITION ⇔ x∈SubsetSum**

so there exist S⊆ {1,...,m,m+1} s.t T = {1,...,m,m+1} - S and Σai (i∈T)= [a+(2t-a)-t]=t

and it shows Σai(i∈T) = Σaj(j∈S) so m+1∈T and S⊆ {1,· · ·,m} and Σai(i∈S)= t

so x∈SubsetSum

*Case 2: 2t =< a* :

we can check same but just this time am+1 is a−2t

edited Apr 20 at 2:41

answered Apr 20 at 2:26

Behrooz Tabesh
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