Why should we accept the preference axioms?

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Outline

1. Why should we accept the preference axioms?
   - Must a rational preference be transitive?
   - Must a rational preference be complete?
   - The multi-attribute approach
   - Must a rational preference satisfy the independence axiom?
   - Risk aversion

The preference axioms

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Transitivity	if $x \succ y$ and $y \succ z$, then $x \succ z$
Completeness	$x \succ y$ or $y \succ x$ or $x \backsim y$
Independence	if $x \succ y$, then $xpz \succ ypz$ (where xpz is a lottery that gives you x with probability p and z with probability $1 - p$.)

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Pragmatic arguments provide the best support currently available for the axioms of decision theory.

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Imagine:
A friend offers to give you exactly one of her three love novels, x or y or z. You feel that you prefer x to y and y to z.

Does it follow from this that you must also prefer x to z?

The transitivity axiom entails that the answer should be affirmative.

money-pump argument outlined:

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- Now suppose that you are in possession of z, and that you are invited to swap z for y.
- Since you prefer y to z, rationality obliges you to swap.

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- So you swap, and temporarily get y.

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- You are then invited to swap y for x, which you do, since you prefer x to y.

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- Finally, you are offered to pay a small amount, say one cent, for swapping x for z.

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- Finally, you are offered to pay a small amount, say one cent, for swapping x for z.
- Since z is strictly better than x, even after you have paid the fee for swapping, rationality tells you that you should accept the offer.

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- So you swap, and temporarily get y.
- You are then invited to swap y for x, which you do, since you prefer x to y.
- Finally, you are offered to pay a small amount, say one cent, for swapping x for z.
- Since z is strictly better than x, even after you have paid the fee for swapping, rationality tells you that you should accept the offer.

This means that you end up where you started, the only difference being that you now have one cent less. This procedure is thereafter iterated over and over again. After a billion cycles you have lost ten million dollars, for which you have got nothing in return.

Outline

Can the completeness axiom be justified by some pragmatic argument?

An influential argument against the completeness axiom. This is the so-called *small improvement argument*.

The small improvement argument is not uncontroversial.

Outline

The multi-attribute approach

The multi-attribute approach seeks to avoid the criticism that money and human welfare are incommensurable by giving up the assumption that all outcomes have to be compared on a common scale.

Example

Rachel has somehow divided the relevant objectives of her decision problem into a list of attributes.

	Attribute 1	Attribute 2	Attribute 3	Attribute 4
Alt a_1	1	3	1	2
Alt a_3	3	1	3	1
Alt a_3	2	2	2	2

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The independence axiom

Can this axiom be justified by a pragmatic argument?

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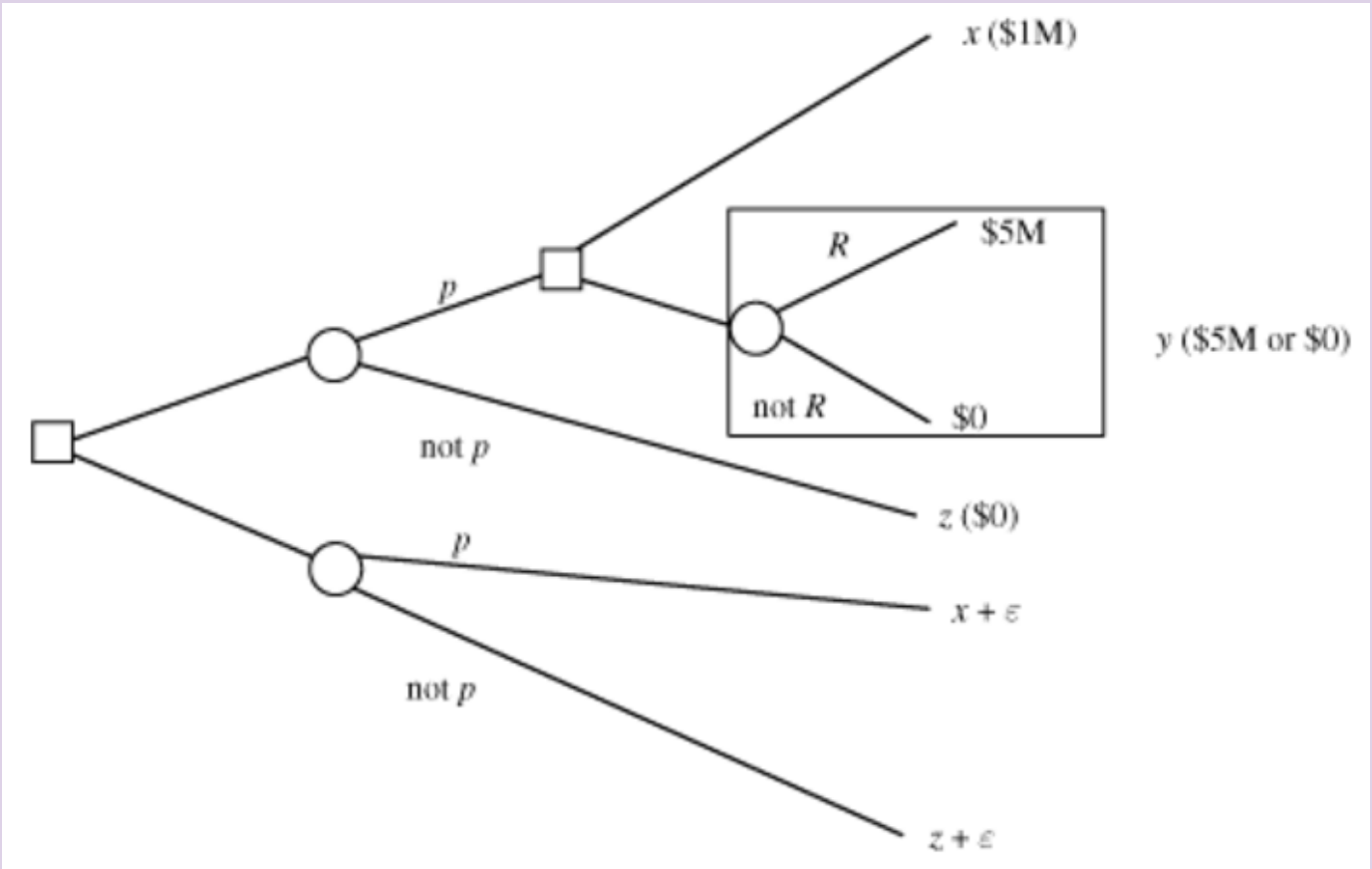
Allais' example:

x	\$ 1,000,000
y	\$5,000,000 iff it rains tomorrow (R), and \$0 otherwise($\neg R$)
z	\$0
$x + \varepsilon$	\$1,000,000 plus one cent
$z + \varepsilon$	\$0 plus one cent

After Rabinowicz 1995



Outline

Discussion of risk aversion

To say that a decision maker is risk averse?

The three most important notions of risk aversion:

- Aversion against actuarial risks
- Aversion against utility risks
- Aversion against epistemic risks