Subjective probability

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- Probability is a kind of mental phenomenon.
- Probabilities are not part of the external world.
  - They are entities that human beings somehow create in their minds.
- This should not be taken to mean that any subjective degree of belief is a probability.

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- The key idea in modern subjective probability theory (Ramsey, de Finetti and Savage) is to introduce an ingenious way in which subjective probabilities can be measured.
- The measurement process is based on the insight that the degree to which a decision maker believes something is closely linked to his or her behavior.

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- We say that f and g agree with each other in the set of states B if and only if $f(s) = g(s)$ for all $s \in B$.

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  (2) $u' = c \cdot u + d$.

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  A You win $100 if Bond manages to disarm the bomb and nothing otherwise.
  B You win nothing if Bond manages to disarm the bomb and $100 if the bomb goes off.

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- A natural reaction to the James Bond problem is to argue that one should simply add the assumption that utilities have to be state-independent.
- Then the James Bond example could be ruled out as an illegitimate formal representation of the decision problem, since the utility of money seems to be state-dependent.

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- If the three states denote three possible exchange rates between dollars and yen, this would render the decision maker's preferences perfectly coherent.

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- *(de Finetti's part) If a player's betting quotients violate the probability axioms, then she can be exploited in a Dutch Book that leads to a sure loss.*

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    - No utility function is derived; de Finetti simply took for granted that the decision maker's utility of money and other goods is linear. Many scholars have pointed out that this is a very strong assumption.

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- The formula $X \succcurlyeq Y$ is an abbreviation for 'either $X \succ Y$ or $X \sim Y$, but not both'.

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QP 2 For any two events X and Y, exactly one of the following three relations hold: $X \succ Y$, or $Y \succ X$, or $X \sim Y$.

QP 3 If X_1, X_2, Y_1 and Y_2 are four events such that $X_1 \cap X_2 = Y_1 \cap Y_2 = \varnothing$ and $Y_i \succcurlyeq X_i$ for $i = 1, 2$, then $Y_1 \cup Y_2 \succcurlyeq X_1 \cup X_2$. If, in addition, either $Y_1 \succ X_1$ or $Y_2 \succ X_2$, then $Y_1 \cup Y_2 \succ X_1 \cup X_2$.

Minimal subjectivism

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QP 5 There exists a (subjective) random variable which has a uniform distribution on the interval $[0, 1]$.

Minimal subjectivism

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- *QP 1-5 are jointly sufficient and necessary for the existence of a unique function p*

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- *QP 1-5 are jointly sufficient and necessary for the existence of a unique function p that assigns a real number in the interval $[0, 1]$ to all elements in E,*

Minimal subjectivism

Theorem 5

- *QP 1-5 are jointly sufficient and necessary for the existence of a unique function p that assigns a real number in the interval* $[0, 1]$ *to all elements in E, such that* $X \succcurlyeq Y$ *if and only if* $p(X) \geq p(Y)$. *In addition, p satisfies Kolmogorov's axioms.*

Minimal subjectivism

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Lemma 6

Minimal subjectivism

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- *If x is any element in E, then there exists a unique number a^* $(1 \geq a^* \geq 0)$ such that $x \sim G[0, a^*]$.*