Game Theory - Homework I

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1 Instructions

- 1. The homework is due on September 15, in class.
- 2. Each question is worth 4 points.
- 3. The Bonus problem is worth 5 points.
- 4. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

- 1. Consider the following version of chess, in which there are three possible outcomes:
 - (a) Victory for White, if White captures the Black King,
 - (b) Victory for Black, if Black captures the White King.
 - (c) A draw, if:
 - i. if Black is stalemated,
 - ii. if White is stalemated,
 - iii. a position repeats.

Argue informally (or formally) that one and only one of the following must be true:

- (a) White has a winning strategy.
- (b) Black has a winning strategy.
- (c) Each of the two players has a strategy guaranteeing at least a draw.

You may assume that chess is a finite game.

2. Consider the following bargaining problem: Two players must divide one unit of a perfectly divisible good into fractions α and β , such that $\alpha, \beta \ge 0$ and $\alpha + \beta = 1$. If they cannot agree to a split, they get nothing. Player 1 has the utility function $u_1(\alpha) = \alpha$ and player 2 has the utility function $u_2(\alpha) = \sqrt{\alpha}$. How would you split the good so as to maximize the product of the player's utilities?

3. Consider the following zero-sum game:

	Left	Right	
Up	3	2]
Down	1	4	

Find a Nash equilibrium for this game.

- 4. Consider the roulette game in Vegas, where you bet on a number and the probability that you win is $\frac{1}{38}$, since there are 38 equi-probable outcomes. If the ball lands on the number you have chosen, you will be paid 35 times the amount that you bet and the bet itself.
 - (a) Formalize and visualize the decision problem in a decision matrix.
 - (b) Formalize and visualize the decision problem in a decision tree.
 - (c) How much can you expect to win on average, for every dollar you bet?
- 5. Explain in as much detail as you can (with examples), the differences between:
 - (a) Ordinal scales and Cardinal scales, and
 - (b) Interval scales and Ration scales.

BONUS: Briefly summarize the major events in the history of decision theory.