Game Theory - Homework II

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1 Instructions

1. The homework is due on October 13, in class.
2. Each question is worth 4 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. Consider the following decision problem:

Act \ Payoff				
a_1	\$20	\$20	\$0	\$10
a_2	\$10	\$10	\$10	\$10
a_3	\$0	\$40	\$0	\$0
a_4	\$10	\$30	\$0	\$0

   How would you choose your action as per the following rules:

   (a) Maximin,
   (b) Maximax,
   (c) Minimax regret,
   (d) Optimism-Pessimism rule (with $\alpha > \frac{1}{4}$),
   (e) The principle of insufficient reason.

2. A hospital has budgeted $\$480,000$ to build in-patient and out-patient facilities. Each in-patient facility costs $\$12,000$ to build and each out-patient facility costs half that. The fire and planning authorities have mandated that the hospital should accommodate at most 60 patients. The hospital management wants to maximize revenue which consists of fees charged to patients in both facilities. What is the optimal balance of in-patients and out-patients that should be admitted to the hospital and what are their implications to the fees charged to in-patients and out-patients?

3. (a) Assume you play a game in which the the jackpot is \$1 million and your utility of money is given by: $u(x) = \ln(x+1)$. Further assume that you are only interested in maximizing expected utility.
      i. How much should you pay to play the game, if your chances of winning are 1 in a million?
      ii. How much should you pay to play the game, if your chances of winning are 1 in a thousand?
      iii. Can you explain the infinitesimal difference in the two payments?

(b) Consider the following decision problem:

State (probability) / Act	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
a_1	\$49	\$25	\$25
a_2	\$36	\$100	\$0
a_3	\$81	\$0	\$0

i. Which act is to be preferred if the sole goal is to maximize expected monetary value?

ii. Assume that the decision maker's utility is given by $u(x) = \sqrt{x}$. Which act is to be preferred if the goal is to maximize expected value?

4. Consider the following zero-sum game:

$$
\begin{array}{c}
\\ Up \\ Middle \\ Down
\end{array}
\begin{array}{c}
\begin{array}{ccc} Left & Center & Right \end{array} \\
\begin{bmatrix} 3 & -3 & 0 \\ 2 & 6 & 4 \\ 2 & 5 & 6 \end{bmatrix}
\end{array}
$$

(a) Find a mixed strategy for Player I (Row player) which guarantees him the same payoff against any pure strategy of Player II?

(b) Find a mixed strategy for Player II which guarantees him the same payoff against any pure strategy of Player I?

(c) Argue that the two strategies discovered thusly are in fact the optimal strategies of the two players.

5. (a) Suppose that Ram is indifferent to the following two lotteries: Getting A for certain and getting B with 0.9 probability and C with probability 0.1. Also suppose that Ram is indifferent to the following two lotteries: Getting A for certain and getting B with probability 0.6 and D with probability 0.4. Assume that Ram's preferences satisfy the von Neumann-Morgenstern axioms.

i. Does Ram prefer C or D?

ii. What is the relative difference in utility between B and C and between B and D?

iii. Assuming that Ram's utility of B is 1 and his utility of C is 0, what are his utilities of B and D?

(b) Consider the following two lotteries: In Lottery A, you get \$100 with 0.5 probability and \$10 with 0.5 probability. In Lottery B, you get \$200 with probability 0.25, \$50 with probability 0.25 and \$10 with probability 0.5. Assume that you prefer Lottery A to Lottery B.

Now consider the following two lotteries: In Lottery C, you get \$200 or \$50 with 0.5 probability. In Lottery D, you get \$100 for certain. Assume that you prefer Lottery C to Lottery D.

Are your preferences consistent with the Morgenstern-von Neumann axioms? You need to discuss each axiom with respect to your preferences.

(c) Show that if a relation $\succ$ is asymmetric and negatively transitive, then it is transitive.

(d) Show that negative transitivity is logically equivalent to the following claim:

$$(\forall x)(\forall z)(x \succ z) \rightarrow [(\forall y)(x \succ y) \vee (y \succ z)].$$