Game Theory - Homework III

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1 Instructions

1. The homework is due on November 15, in class.
2. Each question is worth 4 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. (a) Four objects O_1, O_2, O_3 and O_4 have different worths for two players 1 and 2 as given by the following table:

	O_1	O_2	O_3	O_4
Player 1 Worth:	1	2	3	4
Player 2 Worth:	2	3	4	1

   Player 1 starts by choosing an object. Player 2 then chooses an object. This is followed by Player 1 picking his second object and then Player 2 finally picking up the object that is left.

   i. Draw the decision tree for this extensive form game.
   ii. How many strategies does each player have?
   iii. Determine the sublime perfect equilibria in pure strategies.

   (b) Assume that two hoteliers, Jeff and Bob, are looking to open a hotel each on a beach strip represented by $[-a, a]$. Further assume that the hoteliers can pick any point on this continuous interval to open their hotel. The demand along the strip is represented by the uniform distribution $U(a)$.

   i. Assume that Jeff choses to open his hotel at point $p > 0$. What is Bob's best response?
   ii. Find the unique Nash equilibrium of this game.
   iii. How would the Nash equilibrium change, if the demand distribution is not uniform? Explain with an example.

2. (a) A gambler has two coins in his pocket, a fair coin and a 2-headed coin. He chooses one of the coins uniformly and at random and tosses it. It shows up heads. What is the probability that the fair coin was chosen? He tosses the same coin a second time and it shows up heads again. What is the probability that the fair coin was chosen? He tosses the coin a third time and it shows up tails. What is the probability that the fair coin was chosen?

   (b) Ten numbered coins are in a box, such that if the i^{th} coin is flipped, it shows up heads with probability $\frac{i}{10}$, $i = 1, 2, \ldots, 10$. One of the coins is selected uniformly and at random and tossed. It shows up heads. What is the probability that it is the fifth coin?

(c) Assume that four students from the class, viz., Zola, Piotr, Geoff and Michael are witnesses in a court proceeding. Furthermore, assume that each one of them tells the truth with probability $\frac{1}{3}$ independently. Zola affirms that Piotr denies that Geoff declares that Michael is a liar. What is the (conditional) probability that Michael is telling the truth?

3. (a) Does cyclicity in preferences mean lack of transitivity? Explain.

   (b) Explain why Samuelson's theory of revealed preferences is incompatible with the small improvement argument?

   (c) Ram's utility for money is $u(x) = x^{\frac{1}{4}}$ and Shyam's utility for money is $u(x) = x^{\frac{1}{5}}$, where x is the current balance in their respective accounts. Who among the two is more averse to actuarial risks for large amounts of money?

4. (a) In Newcomb's problem, assume that box B_1 contains \$0, instead of \$1000. Do major decision principles still conflict? Explain.

   (b) Suppose that $p(X \mathbin{\Box\!\!\rightarrow} Y) = p(\neg X \mathbin{\Box\!\!\rightarrow} Y) = 0$. What can you conclude about Y?

   (c) Suppose that $p(X \mathbin{\Box\!\!\rightarrow} Y) = p(\neg X \mathbin{\Box\!\!\rightarrow} Y) = 1$. What can you conclude about Y?

   (d) Suppose that $p((X \mathbin{\Box\!\!\rightarrow} Y) \mid X) = p((\neg X \mathbin{\Box\!\!\rightarrow} Y) \mid \neg X) = 0$. What can you conclude about Y?

   (e) Suppose that $p((X \mathbin{\Box\!\!\rightarrow} Y) \mid X) = p((\neg X \mathbin{\Box\!\!\rightarrow} Y) \mid \neg X) = 1$. What can you conclude about Y?

5. (a) Explain the term "Bayesianism" in detail.

   (b) Ram's subjective degree of belief in B is 0.1 and his degree of belief in A given B is 0.4. He then comes to believe that the unconditional probability of A is 0.9. What is $p(B \mid A)$ from Ram's perspective?

   (c) Assume that Zola prefers x to z and the lottery $y\,p\,w$ to $x\,p\,w$. If she is a Bayesian, what should her preference be with respect to y and z? If she prefers z to y, should any of her preferences be revised?