# Game Theory - Homework IV

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## 1 Instructions

1. The homework is due on December 6, in class.

2. Each question is worth 4 points.

3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

## 2 Problems

1. Consider the following game: Michael and Zola each simultaneously extend one or two fingers and call out a number. The player whose call matches the total number of extended fingers gets that many pennies from his opponent. If both players call out the same number, the game is a tie and money does not exchange hands.

    (a) Represent the game in normal form, with Zola as the row player and Michael as the column player.

    (b) Is the game matrix symmetric? Suggest a strategy for Michael, assuming that Zola's strategy is to pick one of her strategies uniformly and at random.

2. Find all equilibria in the following games:

    (a) Game $I$:

    |       | $C1$     | $C_2$  |
    |-------|----------|--------|
    | $R_1$ | $-3, -3$ | $3, 0$ |
    | $R_2$ | $0, 3$   | $1, 1$ |

    (b) Game $II$:

    |       | $C1$   | $C_2$  |
    |-------|--------|--------|
    | $R_1$ | $0, 3$ | $4, 2$ |
    | $R_2$ | $6, 2$ | $3, 3$ |

3. (a) Let $\{a, b, c, d, e\}$ denote a set of social states. Assume that Zola, Michael, Piotr and Marcela have the following preferences:

   - Zola - $a \succ b \succ c \succ d \succ e$.
   - Michael - $b \succ a \succ c \succ e \succ d$.
   - Piotr - $a \succ c \succ d \succ b \succ e$.

- Marcela - $a \succ b \succ c \succ d \succ e$.

Which state would be selected by traditional voting procedures? How about if the maximin rule were used?

(b) Arrow has argued that social choice cannot be based on majority voting procedure. Explain in detail whether or not you agree with him.

4. (a) Explain the difference between the betting approach to subjective probability and the qualitative approach taken by DeGroot. Mention the merits and demerits of each approach.

(b) Explain the notion of a Dutch book. Construct a Dutch book to show that if $A$ is impossible, then $p(A) = 0$.

(c) Explain what the sure-thing principle is and why it is controversial.

5. (a) Consider the following game:

|   | $X$ | $Y$ | $Z$ |
|---|-----|-----|-----|
| $A$ | $1,1$ | $0,0$ | $2,0$ |
| $B$ | $1,2$ | $1,2$ | $1,1$ |
| $A$ | $0,0$ | $1,1$ | $1,1$ |

Determine the Nash equilibria by using the IEWDS strategy discussed in class. Does the strategy present a problem?

(b) Explain with examples the differences between Nash equilibria and Evolutionary Stable Strategies. In which context would you use the former and in which context, the latter?