# CS 791 - Network Optimization

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## 1 General Information

1. Meeting Times: Tu-Th, 9:30 $am$ − 10:45 $am$
2. Location: 109, MRB
3. Contact Information: 749 ESB, ksmani@csee.wvu.edu
4. Office Hours: MW, 09:00-10:00 $am$.
5. Textbook - [Ber98] is the course text, although [AMO93] is strongly recommended for supplementary reading.
6. URL - `http://www.csee.wvu.edu/~ksmani/courses/sp07/nopt/nopt.html`
7. Assessment:
   (a) Homework Assignments (2) - You will be handed a homework on February $8$, due on February $20$, and a second homework on March $15$, due on March $27$. Each homework is worth $30\%$ (for a total of $60\%$) of your grade.
   (b) Presentation - You are required to work on a research project of your choosing, either individually or in a group. The research project involves in-depth examination of a problem, possible suggestions for improvement, presentation of the work and a write-up. The project will be worth $40\%$ of your grade.
   (c) A maximum of $5$ bonus points will be awarded for class performance
8. Grade Boundaries
   (a) **A**: $75$ and up
   (b) **B**: $60 - 74$
   (c) **C**: $50 - 59$
   (d) **D**: $45 - 49$
   (e) **F**: $0 - 44$
9. Grading policy - If you have any questions about the grading, you must contact the instructor within two days of your paper being returned.
10. Makeup Policy - If for some reason, you are unable to attend a test or an exam, please meet me at the earliest and I will set an alternate date.

11. **Course Objectives** - The objectives of this course are as follows:
    (a) Introduce rigorous algorithmic analysis for problems in Network Optimization.
    (b) Introduce problems such as shortest paths within the min-cost flow framework.
    (c) Introduce the Network Simplex approach.
12. **Learning Outcomes** - Upon successful completion of this course, students will be able to:
    (a) Apply Flow techniques to the design of network optimization problems.
    (b) Appreciate duality in Network design.
    (c) Apply Network Simplex techniques to network design.
    (d) Develop algorithms for flow variants.

# 2 Syllabus Sketch and Weekly Schedule

## 2.1 Introduction

Paths and Cycles, Flow and Divergence, Path Flows and Conformal Decomposition, The Minimum Cost Flow Problem, Network Flow Problems with Convex Cost, Multicommodity Flow Problems, Discrete Network Optimization Problems, Primal Cost Improvement Algorithms, Dual Cost Improvement Algorithms, Auction Algorithms, Good, Bad, and Polynomial Algorithms. (2 Lectures.)

## 2.2 Shortest Path Problems

Problem Formulation and Applications, A Generic Shortest Path Algorithm, Label Setting (Dijkstra) Methods, Label Correcting Methods, Comparison of Label Setting and Label Correcting, Single Origin/Single Destination Methods, Multiple Origin/Multiple Destination Methods. (3 Lectures.)

## 2.3 The Max-Flow Problem

The Max-Flow and Min-Cut Problems, Cuts in a Graph, The Max-Flow/Min-Cut Theorem, The Maximal and Minimal Saturated Cuts, Decomposition of Infeasible Network Flow Problems, The Ford-Fulkerson Algorithm, Price-Based Augmenting Path Algorithms. (3 Lectures.)

## 2.4 The Min-Cost Flow Problem

Transformations and Equivalences, Setting the Lower Flow Bounds to Zeros, Eliminating the Upper Flow Bounds, Reduction to a Circulation Format, Reduction to an Assignment Problem, Interpretation of Constraint Sensitivity and the Dual Problem, Duality and Constraint Sensitivity for Nonnegativity Constraints. (3 Lectures.)

## 2.5 Simplex Methods for Min-Cost Flow

Using Prices to Obtain the In-Arc, Obtaining the Out-Arc, Dealing with Degeneracy, The Basic Simplex Algorithm, Termination, Properties of the Simplex Method, Initialization of the Simplex Method, Extension to Problems with Upper and Lower Bounds. (2 Lectures.)

## 2.6 Dual Ascent Methods for Min-Cost Flow

Dual Ascent, The Primal-Dual (Sequential Shortest Path) Method, The Relaxation Method, Sensitivity Analysis.
(2 Lectures.)

### 2.7 Auction Algorithms for Min-Cost Flow

The Auction Algorithm for the Assignment Problem, Extensions of the Auction Algorithm, The Preflow-Push Algorithm for Max-Flow, The $\epsilon$-Relaxation Method, The Auction/Sequential Shortest Path Algorithm. ($3$ Lectures.)

### 2.8 Nonlinear Network Optimization

Separable Convex Problems Problems, Multicommodity Flows, Side Constraints, Integer Constraints, Networks with Gains, Optimality Conditions, Duality, Algorithms and Approximations. ($3$ Lectures.)

### 2.9 Convex Separable Network Problems (Optional)

Convex Functions of a Single Variable, Optimality Conditions, Duality, Dual Function Differentiability, Algorithms for Differentiable Dual Problems, Auction Algorithms, Monotropic Programming. ($2$ Lectures.)

### 2.10 Network Problems with Integer Constraints

Formulation of Integer-Constrained Problems, Branch-and-Bound, Lagrangian Relaxation, Local Search Methods, Rollout Algorithms. ($3$ Lectures.)

## 3 Auxiliary

A portion of some lectures will be devoted towards the discussion of homework solutions.

## 4 Social Justice Statement

West Virginia University is committed to social justice. I concur with that commitment and expect to foster a nurturing learning environment, based upon open communication, mutual respect and non-discrimination. Our University does not discriminate on the basis of race, sex, age, disability, veteran status, religion, sexual orientation, color or national origin. Any suggestions to further such a positive and open environment in this class will be appreciated and given serious consideration. If you are a person with a disability and anticipate needing any type of accommodation in order to participate in this class, please advise me of the same and make appropriate arrangements with Disability Services ($293 - 6700$).

If you feel that you are being treated inappropriately or unfairly in any way, please feel free to bring your concerns to my attention; rest assured that doing so will not prejudice the grading process. In return, I expect you to behave professionally and ethically.

## References


[AMO93] R. K. Ahuja, T. L. Magnanti, and J. B. Orlin. *Network Flows: Theory, Algorithms and Applications*. Prentice-Hall, 1993.

[Ber98] Dimitri P. Bertesekas. *Network Optimization: Continuous and Discrete Models*. Athena Scientific, first edition, 1998.