Automata Theory - Homework I

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1 Instructions

1. The homework is due on September 14, in class.
2. Each question is worth 3 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. A tree is defined as an undirected connected graph without any cycles. Argue that if a tree has n nodes, it must have precisely $(n-1)$ edges. *Hint: Use structural induction.*
2. Let $\Sigma = \{0, 1\}$ denote an alphabet. Enumerate five elements of the following languages:
   (a) Even binary numbers,
   (b) The number of zeros is not equal to the number of ones in a binary string.
   (c) The number of zeros is exactly one greater than the number of ones.
3. Let $\Sigma = \{0, 1\}$. The language L_3 is defined as follows:
   $L_3 = \{x \mid x \in \Sigma^*,\ x\ mod\ 3 \equiv 0,\ \text{when interpreted as a number in binary}\}$.
   Is L regular? Justify your answer with a proof or a counterexample.
4. Let L_1 and L_2 denote two languages over an alphabet¡ Σ. For any language $L \subseteq \Sigma^*$, the language L^R consists of those strings in Σ^*, whose reverses are in L. Prove or disprove the following claim: $(L_1 \cup L_2)^R = L_1^R \cup L_2^R$.
5. Convert the λ-NFA in Figure (1) into a DFA. Note that the L in the figure represents λ and that $\Sigma = \{0, 1\}$.

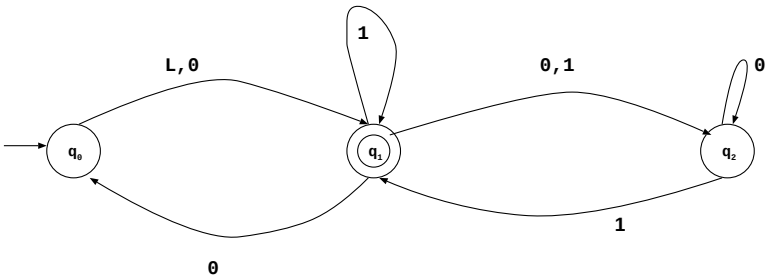

Figure 1: λ-NFA