Automata Theory - Midterm

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1 Instructions

1. The quiz is to be turned in by $12:25$ pm.
2. The quiz is closed-book, although you are permitted one cheat sheet.
3. Each question is worth 4 points.
4. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.
5. The solutions have been posted on the class URL.

2 Problems

1. **Induction:** Consider the context-free grammar $G = \langle V, T, P, S \rangle$, where $V = \{S\}$, $T = \{0, 1\}$, and the productions P are defined by:

$$S \rightarrow 0\,S\,1 \mid 1\,S\,0 \mid S \cdot S \mid \lambda$$

   Argue that every string generated by this grammar is *balanced*, i.e., if w is derived from S, then $n_0(w) = n_1(w)$.

2. **Closure Properties of Regular Languages:** Let L_1 and L_2 be two regular languages. Is the language $L_3 = L_1 \oplus L_2$ regular? Recall that given sets A and B, the set $A \oplus B$ is defined as the set that contains elements which belong to A, but not to B and *vice versa*.

3. **Decision Properties of Regular Languages:** Let L denote a regular language. Describe an efficient decision procedure to test whether $L = L^*$, assuming that the DFA for L is provided.

4. **Proving or Disproving Regularity:** Let $\Sigma = \{a\}$ and let $L = \{a^{n^3},\ n \geq 0\}$. Is L regular?

5. **General questions on Regularity:**
   Let L be a language over some fixed alphabet Σ.
   (a) Assume that L is finite. Is it necessarily regular? Justify your answer. (2 points)
   (b) How would you efficiently test whether $L = \Sigma^*$? (2 points)