Automata Theory - Quiz II

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1 Instructions

1. The quiz is to be turned in by $11:00$ am, November 16.
2. Attempt as many questions as you can. Each question is worth 3 points for full credit; however, you will be given partial credit, as per the policy discussed in class.
3. The solutions will be posted on the class URL.

2 Problems

1. **Induction:** Let L denote the language of balanced strings over $\Sigma = \{0,1\}$, i.e., L consists of all strings in $\{0,1\}$, such that $n_0(w) = n_1(w)$. Given a string $w = w_1 w_2 \ldots w_n$, a *proper prefix* of w is defined to be any string $x = x_1 x_2 \ldots x_k$, such that $k < n$ and $x_i = w_i,\ i = 1, 2, \ldots, k$. Let $y = y_1 y_2 \ldots y_r$ denote an arbitrary string in L with $y_1 = y_r$. Argue that y must have a balanced proper prefix.
2. **Context-Free Grammars:** Consider the grammar $G = \langle V, T, S, P \rangle$, where $V = \{S\}$, $T = \{0,1\}$, $S = S$, and the productions P are defined by:

   $$S \rightarrow 0S1 \mid 1S0 \mid SS \mid \lambda$$

   Argue that $L(G)$ contains the set of *all* balanced strings.
3. **Context-Free Grammar Parsing:** Consider the grammar $G = \langle V, T, S, P \rangle$, with $V = \{S, A, B, C\}$, $T = \{a, b\}$, $S = S$ and the productions P defined by:

   $$\begin{aligned} S &\rightarrow AB \mid BC \\ A &\rightarrow BA \mid a \\ B &\rightarrow CC \mid b \\ C &\rightarrow AB \mid a \end{aligned}$$

   Use the CYK algorithm to establish the membership or non-membership of $w = ababa$ in $L(G)$.
   (You may not use trial and error!)
4. **Pushdown Automaton Design:** Consider the language L defined as follows:
   $L = \{w \in \{0,1\}^* \ :\ n_0(w) = 2 \cdot n_1(w)\}$. Establish that L is context-free by designing a PDA that accepts all and only those strings in L.
5. **Turing Machine Design:** Design a Deterministic Turing Machine to decide the language of palindromes over $\{0,1\}$. You may select the architecture of the Turing Machine, as per your tastes and convenience.