# Automata Theory - Homework I (Solutions)

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## 1 Problems

1. Given sets $R$, $S$ and $T$, show that

$$R \cap (S \cup T) = (R \cap S) \cup (R \cap T)$$

(2 points)

**Proof:** Let $x \in R \cap (S \cup T)$. Then, using the definition of intersection, $x \in R$ *and* $x \in (S \cup T)$. It follows that either ($x \in R$ and $x \in S$) or ($x \in R$ and $x \in T$), which essentially means that $x \in (R \cap S) \cup (R \cap T)$.

Let $x \in (R \cap S) \cup (R \cap T)$. Then, using the definition of union, either $x \in R \cap S$ *or* $x \in R \cap T$. Consider the case that $x \in R \cap S$. It follows that $x \in R$ *and* $x \in S$. However, $x \in S \Rightarrow x \in (S \cup T)$. Therefore, $x \in R \cap (S \cup T)$. The case $x \in R \cap T$ can be argued similarly.

We thus have, $R \cap (S \cup T) = (R \cap S) \cup (R \cap T)$. $\Box$

2. Argue using Mathematical Induction

$$\sum_{i=1}^{n} i^3 = [\frac{(n) \cdot (n+1)}{2}]^2$$

(3 points)

**Proof:** Base case $P(1)$:

$$\begin{aligned}
LHS &= \sum_{i=1}^{1} i^3 \\
&= 1^3 \\
&= 1 \\
RHS &= [\frac{1 \cdot (1+1)}{2}]^2 \\
&= [\frac{1 \cdot (2)}{2}]^2 \\
&= [\frac{2}{2}]^2 \\
&= [1]^2 \\
&= 1
\end{aligned}$$

Thus, $LHS = RHS$ and $P(1)$ is true.

Let us assume that $P(k)$ is true, i.e.

$$\sum_{i=1}^{k} i^3 = [\frac{k \cdot (k+1)}{2}]^2$$

We need to show that $P(k+1)$ is true.

$$\begin{aligned}
LHS &= \sum_{i=1}^{k+1} i^3 \\
&= 1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3 \\
&= [\frac{k \cdot (k+1)}{2}]^2 + (k+1)^3 \quad (using\ the\ inductive\ hypothesis) \\
&= \frac{k^2 \cdot (k+1)^2}{4} + (k+1)^3 \\
&= \frac{k^2 \cdot (k+1)^2 + 4 \cdot (k+1)^3}{4} \\
&= \frac{(k+1)^2 \cdot [k^2 + 4 \cdot (k+1)]}{4} \\
&= \frac{(k+1)^2 \cdot [k^2 + 4k + 4)]}{4} \\
&= \frac{(k+1)^2 \cdot (k+2)^2}{4} \\
&= [\frac{(k+1) \cdot (k+2)}{2}]^2 \\
RHS &= [\frac{(k+1) \cdot ((k+1)+1)}{2}]^2 \\
&= [\frac{(k+1) \cdot (k+2)}{2}]^2
\end{aligned}$$

$LHS$=$RHS$. Thus, we have shown that $P(k) \rightarrow P(k+1)$; applying the principle of mathematical induction, we conclude that the conjecture is true. □

3. Draw the transition diagram for a DFA accepting all strings $x \in \{0,1\}^*$, having 011 as a substring. (2 points)

   **Solution:**

   □

4. Convert the NFA $N =< Q, \Sigma, \delta, q_0, F >$ to a DFA, where

   - $Q = \{p, q, r, s, t\}$,
   - $\Sigma = \{0, 1\}$,
   - $\delta =$
   - $q_0 = p$,
   - $F = \{s, t\}$

   (3 points)

   **Solution:** The above NFA is equivalent to the following DFA $D =< Q, \Sigma, \delta, q_0, F >$, where

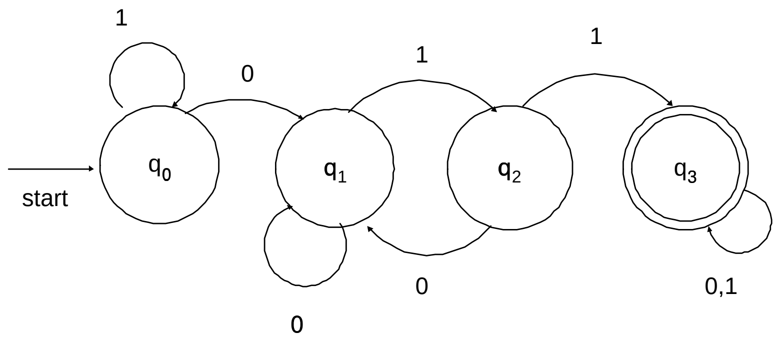


Figure 1: The transition diagram for a DFA accepting all strings $x \in \{0,1\}^*$, having 011 as a substring.

| | 0 | 1 |
|---|---|---|
| p | {p,q} | {p} |
| q | {r,s} | {t} |
| r | {p,r} | {t} |
| s | $\phi$ | $\phi$ |
| t | $\phi$ | $\phi$ |

- $Q = \{\{p\}, \{pq\}, \{pt\}, \{pqrs\}\}$,
- $\Sigma = \{0, 1\}$,
- $\delta =$

| | 0 | 1 |
|---|---|---|
| p | {pq} | {p} |
| pq | {pqrs} | {pt} |
| pt | {pq} | {p} |
| pqrs | {pqrs} | {pt} |

- $q_0 = \{p\}$,
- $F = \{\{pt\}, \{pqrs\}\}$

□