Automata Theory - Homework II (Solutions)

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1 Problems

1. Let L be a regular language with finite alphabet Σ. Suppose that you are given the DFA A representing L. Provide an algorithm to test whether $L = \Sigma^*$, i.e., whether L includes all strings over its alphabet. (3 points)

   Solution: Observe that the question is equivalent to asking whether the complement language L^c is empty. Further, if we switch the accepting and non-accepting states of the DFA A, we get the DFA (say A') that accepts L^c. In this new DFA, i.e., A', use a graph reachability algorithm to check whether *any* final state is reachable from the start state; if so, then there is at least one string in L^c, which means that $L \neq \Sigma^*$. If no final state is reachable from the start state in A', it follows that L^c must be empty, which implies that $L = \Sigma^*$.

   □

2. Construct the minimum-state equivalent DFA for the DFA represented below. (5 points)

	0	1
$\rightarrow A$	B	E
A	B	E
B	C	F
$*C$	D	H
D	E	H
E	F	I
$*F$	G	B
G	H	B
H	I	C
$*I$	A	E

   Solution: The minimum state DFA for the DFA represented above is:

	0	1
$\rightarrow \{A, D, G\}$	$\{B, E, H\}$	$\{B, E, H\}$
$\{B, E, H\}$	$\{C, F, I\}$	$\{C, F, I\}$
$*\{C, F, I\}$	$\{A, D, G\}$	$\{B, E, H\}$

   □

3. Design a CFG for the language $L = \{0^n 1^n | n \geq 1\}$, i.e., the set of all strings of one or more 0's, followed by an equal number of 1's. (2 points)

Solution: A Context-Free Grammar for the language L is:
$S \rightarrow 0S1$
$S \rightarrow 01$ □