Automata Theory - Quiz II (Solutions)

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1 Problems

1. Let L_1 and L_2 be 2 regular languages, over the same alphabet Σ and let them be respectively represented by DFAs A_1 and A_2. Discuss a strategy by which you could check whether there is a string in Σ^*, that is neither in L_1 nor in L_2. (3 points)

   Solution: Observe that we want to test whether M is empty, where $M = L_1' \bigcap L_2'$. This is equivalent to testing whether $M = (L_1 \bigcup L_2)'$ is empty. Thus we can combine DFA's A_1 and A_2 to form an ϵ-NFA (say B), which contains a start state that on transition ϵ goes to both A_1's and A_2's old start states. Similarly, we have a new final state to which all the original final states of A_1 and A_2 are connected. We now must convert B to a DFA (say C). We will then switch the accepting and non-accepting states of C and use the graph reachability algorithm to check whether any accepting state is reachable from the start state, if so, then there is at least one string in M. Otherwise, M is empty. □

2. Consider the grammar $G = (\{S\}, \{a, b\}, P, S)$, where P is defined as the following set of rules:

   $$S \to aSbS \mid bSaS \mid \epsilon$$

   Argue using mathematical induction, that $L(G)$ is the set of *all* strings with an equal number of $a's$ and $b's$. (4 points)

   Proof: Using mathematical induction on the length of the string (say w), we have the following.
   Base Case:
   Let $|w| = 0$, then the production $S \to \epsilon$ derives this string, which has 0 a's and 0 b's.
   Let us assume that $w \in L(G)$ for all $|w| \leq 2^{n-1}$.
   Inductive Step:
   Let $|w| = 2^n$, then w must be of the form $w = aw'b$ or $w = bw'a$. Using the inductive hypothesis, we have $w' \in L(G)$. Now, in order to produce w, we must use either production $S \to aSbS$ or $S \to bSaS$. However, both productions give us an equal number of a's and b's. Thus $|w| = 2^n \in L(G)$.
   Therefore, $L(G)$ is the set of *all* strings with an equal number of $a's$ and $b's$. □

3. Write a CFG for the following language: $L = \{a^i b^j \mid i, j \geq 1,\ j = i + 1\}$. (3 points)

   Solution: A Context-Free Grammar for the language L is:
   $S \to abb$
   $S \to aSb$
   □