

# Automata Theory - Scrimmage I

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## 1 Instructions

1. The Scrimmage will not be graded, i.e., there are no points.
2. Attempt as many problems as you can.

## 2 Problems

1. Prove using Mathematical Induction:

$$\sum_{i=1}^n i^3 = \left(\sum_{i=1}^n i\right)^2$$

2. Let  $\Sigma = \{0, 1\}$ . Draw a DFA for the language  $L$  containing strings having the pattern 010 in them.
3. Let  $\Sigma = \{0, 1\}$ . Draw a NFA for the language  $L$  consisting of strings in which the final digit has appeared before.
4. Repeat the above problem for strings in which the final digit has not appeared before.
5. Let  $\Sigma = \{0, 1\}$ . Let  $L \subseteq \Sigma^*$  represent the language of those strings that do not contain the pattern 101. Argue that  $L$  is regular.
6. Argue that  $(R^*)^* = R^*$ , for any regular expression  $R$ .
7. Is  $L = \{0^n 10^n \mid n \geq 1\}$  regular? Explain.
8. Let  $\Sigma = \{0, 1, 2\}$  and  $\tau = \{a, b\}$ . Consider the homomorphism  $h$  defined by  $h(0) = a$ ,  $h(1) = ab$  and  $h(2) = ba$ . Let  $L$  be the unit string language  $\{ababa\}$ . What is  $h^{-1}(L)$ ?