Automata Theory - Scrimmage II (Solutions)

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1 Problems

1. For $\Sigma = \{a, b\}$, construct a DFA that accepts the set consisting of all strings with no more than 3 a's.

   Solution: □

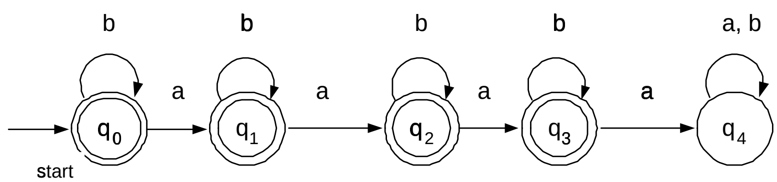

Figure 1: DFA

2. For $\Sigma = \{a, b, c\}$, construct an ϵ-NFA that accepts the language $L = \{ab + abc\}^*$.

   Solution: □

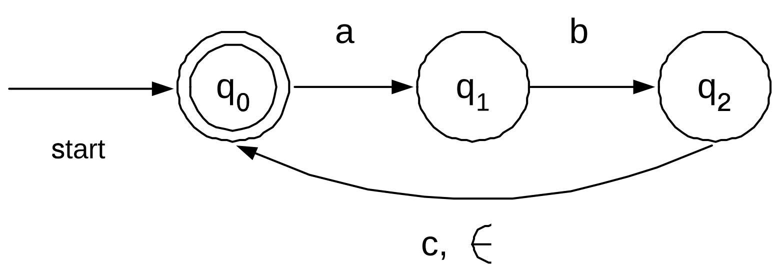

Figure 2: ϵ-NFA

3. Give a regular expression for the following languages.

   (a) $L = \{a^n b^m | n \geq 4, m \leq 3\}$
   Solution: Observe that we can break the solution into the cases $m = 0, 1, 2, 3$. Now, we can write the solution by first generating 4 or more a's followed by the prerequisite number of b's. Thus, the regular expression for L is $aaaaa^*(\epsilon + b + bb + bbb)$. □

   (b) L'
   Solution: Observe that a string is not in L if it is of the form $a^n b^m$, with either $n < 4$ or $m > 3$;

we must also include strings in which a b is followed by an a. Thus, the regular expression for L' is $(\epsilon + a + aa + aaa)b^* + a^*bbbbb^* + (a + b)^*ba(a + b)^*$. □

4. Prove that the following language $L = \{a^n b^l a^k | k \geq n + l\}$ is not regular.

   Proof:

   (a) Player 1 picks the language L to be proved nonregular, where $L = \{a^n b^l a^k | k \geq n + l\}$.

   (b) Player 2 picks n.

   (c) Player 1 picks $w = a^n b^n a^{2 \cdot n}$.

   (d) Player 2 breaks w into xyz, in which $y \neq \epsilon$ and $|xy| \leq n$.

   (e) Player 1 wins. Since $|xy| \leq n$ and xy comes at the front of w, we know that x and y consist of only a's. Thus, $y = a^k$ for $0 < k \leq n$, since $y \neq \epsilon$. The Pumping Lemma tells us that xy^kz is in L if L is regular. If we choose $k = 2$, the resulting string is $w' = a^{n+2}b^n a^{2 \cdot n}$. Clearly w' is not in L. Therefore, we have contradicted our assumption that L is regular.

   □

5. Design a context-free grammar for the language $L = \{a^n b^m | 2 \cdot n \leq m \leq 3 \cdot n, n \geq 0, m \geq 0\}$.

   Solution: The following rules define the context-free grammar.

   (a) $S \rightarrow \epsilon$

   (b) $S \rightarrow aSbb$

   (c) $S \rightarrow aSbbb$

   □