Automata Theory - Scrimmage III (Solutions)

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1 Problems

1. Minimize the states in the DFA depicted in the following diagram.

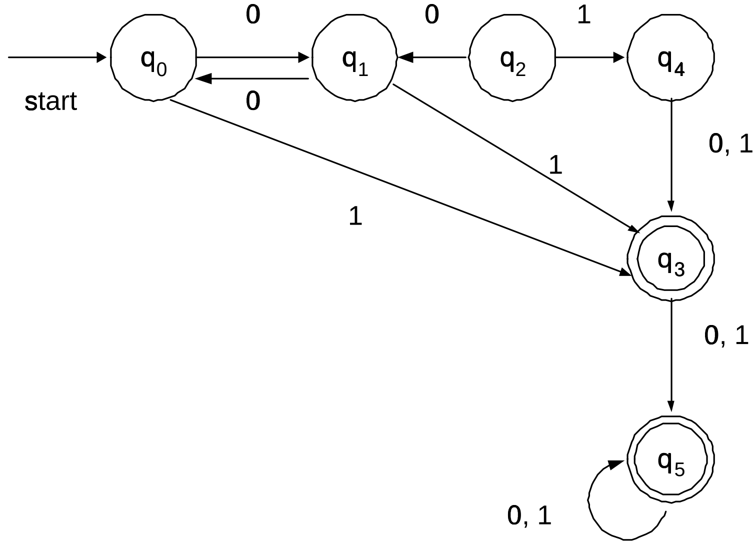

Figure 1: DFA

Solution: □

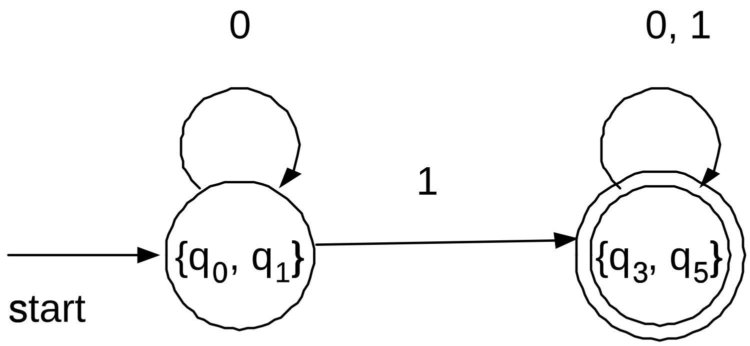

Figure 2: DFA

2. Convert the following regular expression to an NFA $a + b^n$ where $n \geq 1$.

   Solution: □

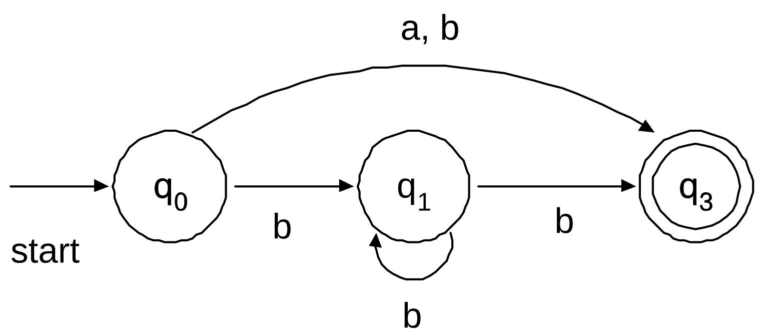

Figure 3: NFA

3. Convert the following ϵ-NFA into an equivalent DFA.

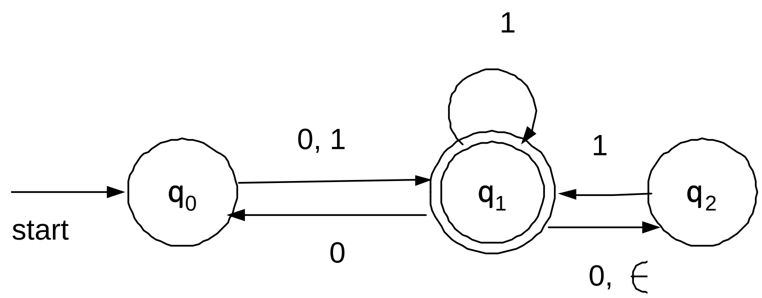

Figure 4: ϵ-NFA

Solution: $\square$

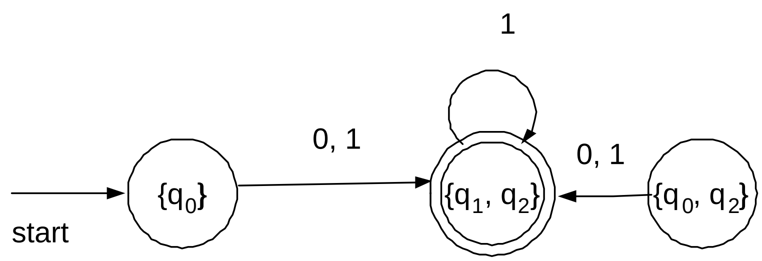

Figure 5: DFA

4. Give a grammar for the following language $L = \{a^n b^m | n + m \;\; is \;\; even\}$.

   Solution:
   $S \rightarrow aaS|A$
   $A \rightarrow bbA|\epsilon$ $\square$

5. Give a PDA that accepts the following language $L = \{a^n b^n | n \geq 0\}$.

   Solution: $P =< Q, \Sigma, \Gamma, \delta, q_0, Z_0, F >$, where

   - $Q = \{q_0, q_1, q_2\}$,
   - $\Sigma = \{a, b\}$,
   - $\Gamma = \{1, Z_0\}$,
   - $F = \{q_0\}$, and
   - $\delta =$

$$
\begin{aligned}
\delta(q_0, a, Z_0) &= \{(q_1, 1Z_0)\} \\
\delta(q_1, a, 1) &= \{(q_1, 11)\} \\
\delta(q_1, b, 1) &= \{(q_2, \epsilon)\} \\
\delta(q_2, b, 1) &= \{(q_2, \epsilon)\} \\
\delta(q_2, \epsilon, Z_0) &= \{(q_0, \epsilon)\}
\end{aligned}
$$

   $\square$

6. Design a Turing Machine that accepts the language defined by the regular expression $a(a + b)^*$. Assume that $\Sigma = \{a, b\}$.

Solution: $M =< Q, \Sigma, \Gamma, \delta, q_0, B, F >$, where

- $Q = \{q_0, q_1, q_2\}$,
- $\Sigma = \{a, b\}$,
- $\Gamma = \{a, b, B\}$,
- B is the blank symbol,
- $F = \{q_2\}$, and
- $delta =$

$$\begin{array}{rcl}
\delta(q_0, a) & = & (q_1, a, R) \\
\delta(q_1, a) & = & (q_1, a, R) \\
\delta(q_1, b) & = & (q_1, b, R) \\
\delta(q_1, B) & = & (q_2, B, R)
\end{array}$$

□