Computational Complexity - Final

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1 Instructions

1. Attempt as many problems as you can. You will be given partial credit.
2. This exam is open book.
3. Feel free to quote any theorem from [HS01]
4. The best 6 answers will count towards your grade.

2 Problems

1. Show that `POLYLOGSPACE` $\neq$ `P`. (6 points)
2. The tautology (TAUT) problem is defined as follows: *Given a Boolean formula ϕ on the variables $\{x_1, x_2, \ldots, x_n\}$, is it the case that ϕ is satisfied by all assignments from the set* $\{\mathbf{true}, \mathbf{false}\}^n$*?* Is TAUT `coNP-complete`, when ϕ is CNF? If yes, provide a proof of `coNP-hardness`; if not provide a polynomial time algorithm for the same. (6 points)
3. Show that `NTIME(`n`)` contains an `NP-complete` language. (6 points)
4. Show that if a language L is disjunctively self-reducible, then $L \in$ `NP`. (6 points)
5. Prove: `NP` $\in$ `E` if and only if for every $L \in$ `NP`, $Tally(L) \in P$. (6 points)

3 Extra Credit

1. Prove that `NP` is not included in `DTIME(`n^k`)`, for any fixed $k \geq 1$. Can we therefore concluded that `P` $\neq$ `NP`? (6 points)
2. Consider the subgraph isomorphism problem: *Given 2 graphs G_1 and G_2, is it the case that G_1 is isomorphic to a subgraph of G_2?* What can you say about the complexity of this problem? (6 points)

References

[HS01] Steven Homer and Alan L. Selman. *Computability and Complexity Theory.* Springer-Verlag, 2001.