Computational Complexity - Midterm

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1 Instructions

1. Attempt as many problems as you can. You will be given partial credit.
2. You can assume that the Program Termination Problem and the Halting Problem are Undecidable.
3. This exam is open book.
4. Feel free to quote any theorem from the book.
5. Extra Credit problems will add to your total.

2 Problems

1. Show that $\mathtt{NL} \subseteq \mathtt{P}$. (5 points)
2. Let $L = \{< e, x > \mid \exists y,\ \phi_e(x) = y$, i.e., machine M_e when started with x halts with y as output.$\}$. Is L decidable? Explain. (5 points)
3. Let $L = \{< e > \mid M_e$ writes a non-blank symbol at least once when started on a blank tape.$\}$. Is L decidable? Explain. (5 points)
4. Show that $A \leq_m B \Rightarrow A \leq_T B$. (4 points)
5. Prove or disprove: *If A and B are c.e., then so are $A \cup B$ and $A \cap B$.* (3 points)
6. Prof. Roberts invents a Turing Machine abstraction that has 3 possible moves at each step, viz., $\{\leftarrow, \rightarrow, \uparrow\}$, where the $\uparrow$ indicates that the head does not move at all. Demonstrate to Professor Roberts, that his abstraction is not more powerful than the standard Turing Machine. (3 points)
7. Let A and B be disjoint *c.e.* sets. Show that $A \leq_T A \cup B$. What happens if A and B are not disjoint? (5 points)

3 Extra Credit

1. Show that $\exists\ A, B\ [A \leq_T B$ *and* $A \not\leq_m B]$. (6 points)