Computational Complexity - Quiz I

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1 Instructions

1. Attempt as many problems as you can. You will be given partial credit.
2. You can assume that the Program Termination Problem and the Halting Problem are Undecidable.

2 Problems

1. Design a Deterministic Turing Machine that accepts the regular language $\mathbf{10}^* + \mathbf{01}^*$, i.e., the set of strings in which the first symbol does not appear again on the input. You may assume that $\Sigma = \{0, 1\}$. Feel free to choose the tape symbols. (4 points)
2. Show that the function that maps a program e to the smallest equivalent program is not Totally Computable. (4 points)
3. Show that every infinite computably enumerable set contains a decidable subset. (2 points)
4. Professor Chikovski has the following algorithm for the Halting Problem: Given a Turing Machine e and a string x, use a Non-deterministic Universal Turing Machine N_u to guess a configuration of the Turing Machine e. If this configuration is a halting configuration for x, N_u declares that e halts on x. If not, N_u declares that e does not halt on x. Has Professor Chikovski solved the Halting problem? (Recall that every Non-deterministic Turing Machine can be simulated by a Deterministic Turing Machine.) (3 points)
5. Prove that: *A set S is computably enumerable if and only if there is a decidable relation $R(x, y)$, such that*

   $$x \in S \Leftrightarrow \exists y \;\; R(x, y).$$

   (3 points)
6. Consider a computer with a 32-bit, 256K RAM memory and a finite control CPU. The memory is partitioned into a Program area and a Data area as shown in Figure (1).

   Assume that a word w is written in the Data Area and a program p is written in the data area; also assume that the finite control can simulate p on w. The finite control has been programmed to track the the contents of the entire memory in one step. What can you say about the following problem: Does p halt on w? (You are allowed to reprogram the finite control to suit your needs.) (4 points)

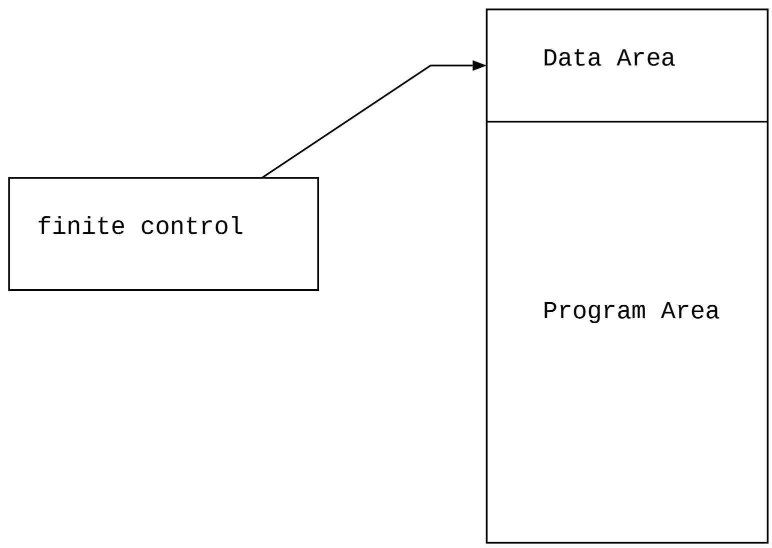

Figure 1: 32-bit computer