Computational Complexity - Quiz II

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1 Instructions

1. Attempt as many problems as you can. You will be given partial credit.
2. You are required to turn in the exam in class on Thursday, April 17.
3. Kindly write up the solutions in LaTeX format.
4. Please refer [HS01], for the definition of complexity classes.
5. You may quote any theorem from [HS01] in your proofs.

2 Problems

1. *Integer Programming Feasibility* is the following problem: Given a system of linear inequalities, $\mathbf{A} \cdot \vec{\mathbf{x}} \leq \vec{\mathbf{b}}$, does there exist an integral solution? Show that Integer Programming Feasibility is `NP-complete`. (4 points)
Hint: Use 3SAT for the reduction.
2. Show that `L = NL`, implies `DLBA = NLBA`. (4 points)
Hint: Use padding techniques.
3. Let $\texttt{ESPACE} = \cup \{\texttt{DSPACE}(k^n) \mid k \geq 1\}$ and $\texttt{NESPACE} = \cup \{\texttt{NSPACE}(k^n) \mid k \geq 1\}$.
Show that $\texttt{ESPACE} = \texttt{NESPACE}$. (4 points)
4. Let $\Sigma = \{0, 1\}$. For any $L \subseteq \Sigma^*$, Tally(L) is defined as: $\{1^{n(w)} \mid w \in L\}$. Note that every string $w \in \Sigma^*$, corresponds to the natural number $n(w)$, which is obtained by treating w as a binary number. Show that Tally(L) $\in \texttt{P}$ implies that $L \in \texttt{E}$. (4 points)
5. Show that $\texttt{LBA} \neq \texttt{P}$ and that $\texttt{DLBA} \neq \texttt{NP}$. (4 points)

References

[HS01] Steven Homer and Alan L. Selman. *Computability and Complexity Theory.* Springer-Verlag, 2001.