Computational Geometry - Homework III

K. Subramani
LCSEE,
West Virginia University,
Morgantown, WV
{ksmani@csee.wvu.edu}

1 Instructions

1. The homework is due on April 27, in class. Each question is worth 5 points.

2. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. **Duality:** Let R be a set of n red points in the plane and B denote a set of n blue points. A line l is said to be a *separator* for R and B, if l has all the red points on one side and all the blue points on the other. Describe a randomized, linear-time algorithm that decides whether the given sets R and B have a separator.

2. **Line Arrangements:** Let L denote a set of n lines in the plane and let $\mathcal{A}(L)$ denote their arrangement. Describe an $O(n \cdot \log n)$ algorithm to compute an axis-parallel rectangle that contains all the vertices of $\mathcal{A}(L)$.

3. **Delaunay Triangulation:** Given a set P of n points in the plane, the Euclidean Minimum Spanning Tree (EMST) is the tree of total edge length connecting all the points in P. Describe an $O(n \cdot \log n)$ algorithm to compute the EMST for P.

4. **Delaunay Triangulation:** Given a set P of n points, the Gabriel graph $\mathcal{G}(P)$ of P is defined as follows: Two points p and q are connected by an edge in $\mathcal{G}(P)$ if and only if the circle with diameter $\bar{pq}$ does not contain any other point of P in its interior.

   (a) Prove that $\mathcal{DG}(P)$ contains $\mathcal{G}(P)$, where $\mathcal{DG}(P)$ denotes the Delaunay graph of P.

   (b) Describe an $O(n \cdot \log n)$ algorithm to compute $\mathcal{G}(P)$.

5. **Geometric Data Structures:** Let I denote a set of n intervals in the plane. Given an interval $r = [x : x']$, we are interested in all the intervals $p \in I$, such that p is completely contained in r. Describe a data structure that uses $O(n \cdot \log n)$ storage and answers these queries in $O(\log n + k)$ time, where k is the number of answers.