Computational Complexity - Final

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1 Instructions

1. The Final is due by $5\ pm$, May 7.
2. Each question is worth 6 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. In class, we showed that a non-deterministic Turing machine is at least as powerful as a randomized Turing machine, which in turn is at least as powerful as a deterministic Turing machine, when it comes to polynomial time and logarithmic space, i.e., **P** $\subseteq$ **RP** $\subseteq$ **NP** and **L** $\subseteq$ **RL** $\subseteq$ **NL**. Does such a relationship hold for problems solvable in polylogarithmic parallel time, using a polynomial number of processors? Explain.
2. The inclusion relationship between **NP** and **BPP** has been an object of intense study among complexity theorists. Argue that if $\mathbf{NP} \subseteq \mathbf{BPP}$, then $\mathbf{NP} = \mathbf{RP}$.
3. Consider the following heuristic for finding the minimum vertex cover in an unweighted, undirected graph.

Function APPROX-VERTEX-COVER($G = \langle V, E \rangle$)
Let $C = \emptyset$.
while ($|E| \neq 0$) **do**
  Pick the vertex $v \in V$ with the largest degree.
  $C = C \cup \{v\}$.
  Delete all edges from E that are incident to v.
end while

Algorithm 2.1: Heuristic for vertex cover

How bad can the approximation ratio of this heuristic get? Contrast with the naive approximation algorithm discussed by Ron in class!

4. The polynomial resolution conjecture states that any unsatisfiable boolean expression has a resolution refutation of polynomial depth. What are the consequences for traditional complexity classes, if this conjecture is proven?
5. Let ϕ be a boolean expression in CNF, over the variables $x_1, x_2, \ldots, x_n$. Consider the problem of answering queries of the form:

$$\forall x_1\ \forall x_2\ \ldots \forall x_n\ \phi$$

What is the complexity of this problem? (*Hint: Polynomial Hierarchy.*)