Computational Complexity - Homework I

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1 Instructions

1. The homework is due on February 12, in class.
2. Each question is worth 3 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. Assume that you are given an instance of the Traveling Salesman Problem (TSP) with n cities and inter-city distances d_{ij}, $i, j = 1, 2, \ldots n$. Let S denote some subset of the cities, excluding city 1 and let $C[S, j]$ denote the shortest path that starts in city 1, visits all the cities in S and ends in city j.
   (a) Use Dynamic Programming to compute $C[S, j]$, i.e., in computing $C[S, j]$ for a given S, use the values computed for subsets of S.
   (b) Analyze the space and time requirements of your algorithm.
   (c) Modify this algorithm to devise a *polynomial time* algorithm for the problem of computing the shortest path from city 1 to city n; note that this shortest need not visit all the other cities.
2. Argue that if a Turing Machine uses less than $c \log \log n$ space, for all $c > 0$, then it uses constant space.
3. Assume that you have a k-string NDTM (Non-deterministic Turing Machine) that accepts a language L in time $f(n)$. Show that L can accepted by a 2-string NDTM in time $O(f(n))$.
4. Classify each of the following languages (with appropriate justification) as recursive, recursively enumerable (but not recursive), or not recursively enumerable.
   (a) $L = \{\langle M \rangle : M \text{ halts on the empty string}\}$.
   (b) $L = \{\langle M \rangle : M \text{ halts on at least one string}\}$.
   (c) $L = \{\langle M, \ M' \rangle : L(M) = L(M')\}$.
5. Let S be an infinite set of boolean expressions, such that every finite subset of S is satisfiable. Argue that S itself must be satisfiable. i.e., the conjunction of all the expressions in S is satisfiable.