Computational Complexity - Homework I

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1 Instructions

- 1. Each question is worth 4 points.
- 2. You need to turn in the quiz by 9 am on 2/17/2010.

2 Problems

- 1. Consider the set $S = \{f \mid f : N \to N\}$. Is S countable? What can you say as regards computability, if the range of each function is restricted to the set $\{0, 1\}$.
- 2. Show that a directed graph $G = \langle V, E \rangle$ is acyclic if and only of its nodes can be numbered from 1 to n such that all edges go from lower to higher numbers.
- 3. Design a Turing Machine M that takes as input the string x and computes the function $f(x) = \Box x$. Argue using induction that your Turing Machine is correct.
- 4. Let L be a regular language over an alphabet Σ . If L is infinite, argue that there must exist strings $x, y, z \in \Sigma^*$, such that $y \neq \epsilon$ and $xy^i z \in L$, for all $i \ge 0$.
- 5. Let $L = \{ \langle M \rangle : M \text{ halts on } \epsilon \}$. Is L recursive?