Computational Complexity - Homework II

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1 Instructions

1. Each question is worth 4 points.
2. You need to turn in the quiz by $9\ am$ on $3/17/2010$.

2 Problems

1. Classify each of the following languages (with appropriate justification) as recursive, recursively enumerable (but not recursive), or not recursively enumerable.
   (a) $L = \{\langle M \rangle : M \text{ halts on at least one string}\}$.
   (b) $L = \{\langle M,\ M' \rangle : L(M) = L(M')\}$.
2. Assume that you are given a formula ϕ in CNF over the variables $X = \{x_1, x_2, \ldots, x_n\}$. Let $C_1 = (x_1 \vee P)$ and $C_2 = (\bar{x_1} \vee Q)$ denote two clauses in ϕ, where P and Q are arbitrary disjuncts over X. Let $\phi' = \phi \wedge (P \vee Q)$. The above process is called a resolution step on variable x_1.
   (i) Show that resolution steps are solution preserving.
   (ii) We can continue in this fashion resolving on x_1 and other variables, deriving a new formula at each time till no new clauses can be added. Call the final formula ϕ^*. Show that ϕ is satisfiable if and only if ϕ^* does not contain the empty clause ($\Box$). Note that $\Box$ is the resolvent of the clauses (x_i), $(\bar{x}_i)$, for any $i = 1, 2, \ldots, n$.
3. Let Σ_{EG} denote an enhanced vocabulary for graph theory. Σ_{EG} has one function symbol $\langle + \rangle$. It has three relational symbols, viz., the binary relation $=$, a binary relation $<$, and a ternary relation, called G_w. Typical expressions in the new vocabulary are: $G_w(x, y, 5)$, $G_w(y, z, -5)$, $(\forall z)(\exists y)G_w(z, y, \infty)$ and so on. Write a sentence in Σ_{EG} which describes graphs containing negative cost cycles.
4. Let P_1 denote the statement: If Δ is consistent, then Δ has a model. Let P_2 denote the statement: If $\Delta \models \phi$, then $\Delta \vdash \phi$. Argue that P_1 and P_2 are equivalent.
5. Show that for any first-order expression ϕ over the vocabulary Σ_G, the property $\phi-$GRAPHS can be tested in logarithmic space.