Optimal binary search trees

→ What is a binary search tree

→ Full, balanced, depth

→ search-time in a key → C_k

→ avg. search time

→ when keys are probabilities

Mean, Mean, Odeal Draw all search trees

0.7 0.2 0.1

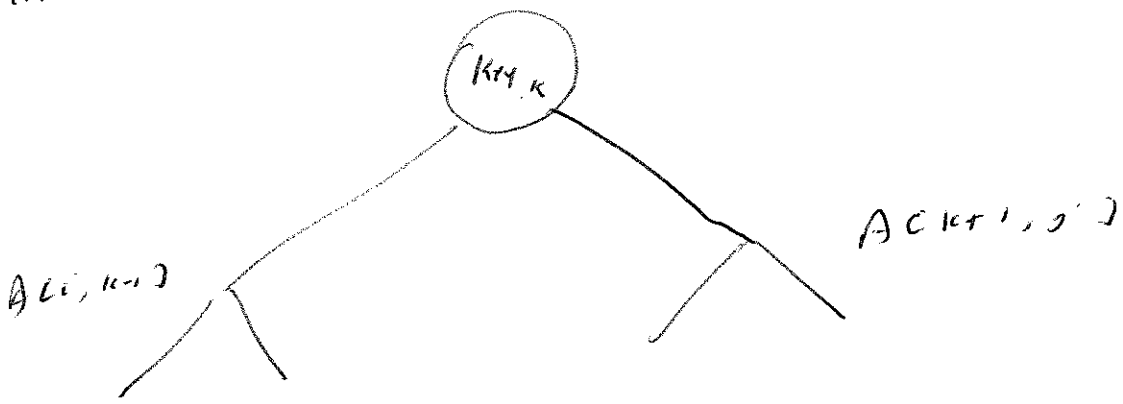
→ Expected time

→ Let $A[i,j]$ denote the optimal/appropriate search tree for keys i ... key j

This search tree minimizes $\sum_{k=i}^{j} C_k p_k$

$A[i,i] = p_i$

→ Principle of optimality



$$A[i,j] = \min_{i \le k \le j} \; A[i,k-1] + A[k+1,j] + p_k + p_{i+1} \cdots p_{k-1} + p_{k+1} \cdots p_j$$

$$= \sum_{k=i}^{j} p_k$$

$O(n^3)$