(1) Difference between prob. anal and and rand. alg

(2) Rand alg → No guarantees but prob. of bad thing is small.

(3) Rand # generator

(4) Hiring Problem → $n$ to $\log n$ , Text descr. different

(4.5) Quicksort algo, partition $j \leftarrow$ Partition $(A, n, q)$,

(5) Worst-case analysis of Quicksort

$$T(n) = \max_{0 \le q \le n-1} \left( T(q) + T(n-q-1) \right) + \Theta(n)$$

Show that $T(n) \le c \cdot n^2$

(6) Best case

(7) Balanced case $T(n) \le T(9n/10) + T(n/10) + cn$

(8) Expected case analysis, randomized partition

$Z = \{ z_1, z_2, \ldots z_n \}$ $\quad Z_{ij} = \{ z_i, z_{i+1}, \ldots z_j \}$

$X_{ij} = 1$, if $z_i$ is compared to $z_j$

$= 0$, otherwise

$\cancel{E[X]}$ $X = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} X_{ij}$

$E[X] = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} p_{ij}$

$Pr[z_i$ and $z_j$ are compared$]$

$= Pr[(z_i$ or $z_j$ is picked$) \cap$ an element of $Z_{ij}$ is picked$]$

$= \dfrac{Pr((z_i \text{ or } z_j \text{ is picked}) \cap \text{an element in } Z_{ij} \text{ is picked})}{Pr(\text{an element in } Z_{ij} \text{ is picked})}$

$= 2/...$

$P_{ij} = 2/_{j-i+1}$

$$\Rightarrow E[X] = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{2}{j-i+1} = \sum_{i=1}^{n-1} \sum_{k=1}^{n-i} \frac{2}{k+1}$$

$$\leq \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{2}{k}$$

$$= O(n \log n)$$

$\longrightarrow$ Blue book analysis

Sorting in Linear Time (Mention)

Medians and Order Statistics

input: $A[1..n]$

output: The element $x \in A$ that is larger than exactly $i-1$ other elements of $A$.

① find min $O(n)$, max $O(n)$, both $(2n-2)$, can be improved to $\frac{3}{2}n-2$, lower bound on min, tournament

Randomized - Select (A, p, r, i)
/* finds the $i^{th}$ smallest element in A[p..r]
if (p >= r)
return A[p]
q ← Random-Partition (A, p, r)
k ← q - p + 1
if i = k
return A[q]
else if i < k
RS (A, p, q-1, i)
else
RS (A, q+1, r, i-k)

A[7, 12 18] $\quad q$

$\stackrel{3}{=} \quad q = 12$

6

p  q
7 8 9 10 11 12 .. 1

Worst-case running time

Expected running time

let A[p..q] have n elements

T(n) is the random variable that denotes the running time on n elements

$X_k = 1$, A[p..q] has exactly k elements
$= 0$, otherwise $\quad 1 \le k \le n$

$E[X_k] = 1/n$

$$T(n) \le \sum_{k=1}^{n} X_k \cdot (T(\max(k-1, n-k)) + O(n))$$

$$= \sum_{k=1}^{n} X_k \cdot T(\max(k-1, n-k)) + O(n)$$

$$E[T(n)] \le \sum_{k=1}^{n} \frac{1}{n} E[T(\max(k-1, n-k))] + O(n)$$

$$\le \frac{2}{n} \sum_{k=n/2}^{n-1} E[T(k)] + O(n)$$

Assume $T(n) \le cn$, $O(n) = an$.

$$E[T(n)] \le \frac{2}{n} \sum_{k=n/2}^{n-1} (ck + an$$

$$= \frac{2c}{n} \left( \sum_{k=1}^{n-1} k - \sum_{k=1}^{n/2-1} k \right) + an$$

$$= \frac{2c}{n} \left( \frac{(n-1)n}{2} - \frac{(n/2-1)(n/2)}{2} \right) + an$$

$$\le \frac{3cn}{4} + \frac{c}{2} + an$$

$$= cn - \left( \frac{cn}{4} - \frac{c}{2} - an \right)$$

We need $\frac{cn}{4} - \frac{c}{2} - an \ge 0$

$$\Rightarrow n \ge \frac{2c}{c-4a}$$

$T(n) = O(1)$, for $n < \frac{2c}{c-4a}$ $\Rightarrow T(n) = O(n)$

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Second analysis