

① Lower bound for comp-based sorting.

- Any comp-based algorithm is a decision tree
- Root is input and leaves are output
- Leaves represent permutations at input
- If alg. is to be correct, all $n!$ permutations should be at the leaves
- An alg. has to take time at least equal to depth of tree $= \Theta(\log n!)$
- Height of tree is $\log n!$

② Deterministic linear-time solution

- Divide into $\frac{n}{5}$ groups of 5 elements each
- Find median of each group
- Recursively find median of the medians
- Use this median to partition array
- Recurse on the appropriate side

$$T(n) \leq O(n) + T\left(\frac{n}{5}\right) + O(n) + T\left(\frac{3n}{10}\right)$$

$$\leq T\left(\frac{n}{5}\right) + T\left(\frac{3n}{10}\right) + Cn$$

$$= O(n)$$

$$\frac{n}{5} \quad \frac{3n}{10}$$

$$\frac{n}{10}$$