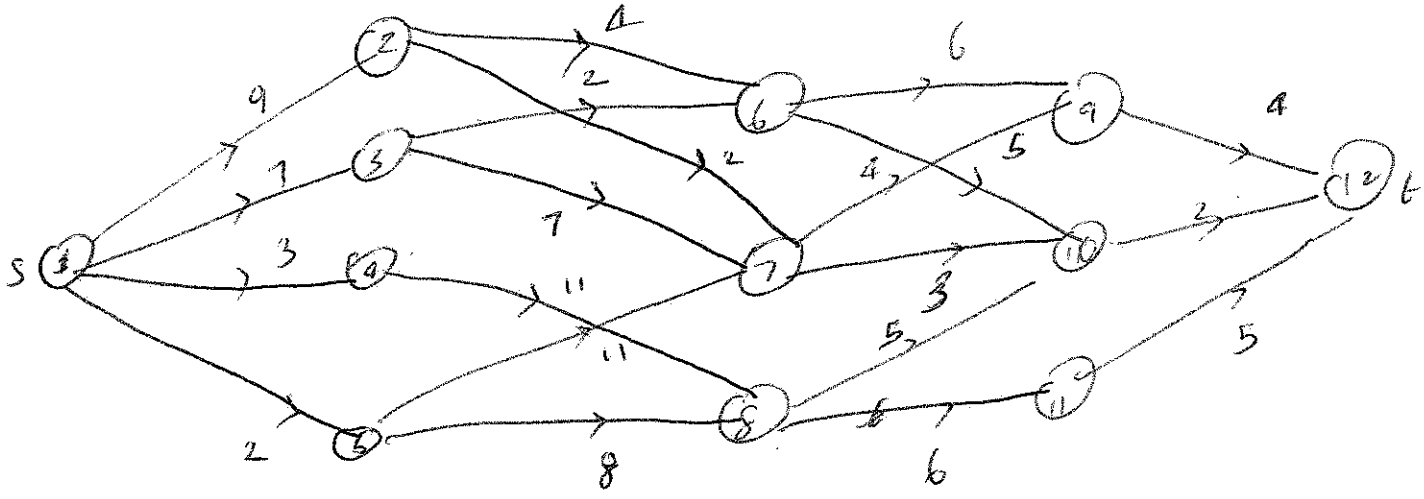
Dynamic Programming

① Multi-stage graphs



K-2 division

Let $P(i,j)$ = min cost path from vertex j in V_i to t and let $cost(i,j)$ be the cost of that path.

$$cost(i,j) = \min_{l \in V_{i+1}, \langle j,l \rangle \in E} \{ c(j,l) + cost(i+1,l) \}$$

cost(4, 9) — 10, 11

cost(3, 6) — 7, 8

cost(2, 2) — 3, 4, 5

cost(1,1)

Rod cutting problem

i	1	2	3	4	5	6	7	8
p_i	1	5	8	9	10	17	17	20

Optimal substructure

$$r_n = \max(p_n, r_1 + r_{n-1}, r_2 + r_{n-2}, \ldots, r_{n-1} + r_1)$$

Simplified description

$$r_n = \max_{1 \le i \le n} (p_i + r_{n-i})$$

Top-down, very inefficient

Runtime $O(n^2)$