Mathematical Induction

K. Subramani[1]

[1]Lane Department of Computer Science and Electrical Engineering
West Virginia University

11 January, 2011

Outline

Outline

Induction

Motivation

Reaching arbitrary rungs of a ladder.

Induction

Motivation

Reaching arbitrary rungs of a ladder.

Note

Can only be applied to a well-ordered domain, where the concept of "next" is unambiguous, e.g., positive integers.

Induction

Motivation

Reaching arbitrary rungs of a ladder.

Note

Can only be applied to a well-ordered domain, where the concept of "next" is unambiguous, e.g., positive integers.

Principle

Assume that the domain is the set of positive integers.

Induction

Motivation

Reaching arbitrary rungs of a ladder.

Note

Can only be applied to a well-ordered domain, where the concept of "next" is unambiguous, e.g., positive integers.

Principle

Assume that the domain is the set of positive integers.

1. $P(1)$ is **true**.

Induction

Motivation

Reaching arbitrary rungs of a ladder.

Note

Can only be applied to a well-ordered domain, where the concept of "next" is unambiguous, e.g., positive integers.

Principle

Assume that the domain is the set of positive integers.

1. $P(1)$ is **true**.
2. $(\forall k)[P(k) \rightarrow P(k+1)]$

Induction

Motivation

Reaching arbitrary rungs of a ladder.

Note

Can only be applied to a well-ordered domain, where the concept of "next" is unambiguous, e.g., positive integers.

Principle

Assume that the domain is the set of positive integers.

1. $P(1)$ is **true**.
2. $(\forall k)[P(k) \rightarrow P(k+1)]$

$P(n)$ is **true**, for all positive integers n.

Induction

Motivation

Reaching arbitrary rungs of a ladder.

Note

Can only be applied to a well-ordered domain, where the concept of "next" is unambiguous, e.g., positive integers.

Principle

Assume that the domain is the set of positive integers.

1. $P(1)$ is **true**.
2. $(\forall k)[P(k) \rightarrow P(k+1)]$

$P(n)$ is **true**, for all positive integers n.

Note

(i) *Showing that* $P(1)$ *is* **true** *is called the basis step.*

Induction

Motivation

Reaching arbitrary rungs of a ladder.

Note

Can only be applied to a well-ordered domain, where the concept of "next" is unambiguous, e.g., positive integers.

Principle

Assume that the domain is the set of positive integers.

1. $P(1)$ is **true**.
2. $(\forall k)[P(k) \rightarrow P(k+1)]$

$P(n)$ is **true**, for all positive integers n.

Note

(i) *Showing that* $P(1)$ *is* **true** *is called the basis step.*

(ii) *Assuming that* $P(k)$ *is* **true**, *in order to show that* $P(k+1)$ *is* **true** *is called the inductive hypothesis.*

Induction (contd.)

Example

Show that the sum of the first n integers is $\frac{n \cdot (n+1)}{2}$.

Induction (contd.)

Example

Show that the sum of the first n integers is $\frac{n \cdot (n+1)}{2}$. Formally, $\sum_{i=1}^{n} i = \frac{n \cdot (n+1)}{2}$.

Induction (contd.)

Example

Show that the sum of the first n integers is $\frac{n\cdot(n+1)}{2}$. Formally, $\sum_{i=1}^{n} i = \frac{n\cdot(n+1)}{2}$.

Induction (contd.)

Example

Show that the sum of the first n integers is $\frac{n\cdot(n+1)}{2}$. Formally, $\sum_{i=1}^{n} i = \frac{n\cdot(n+1)}{2}$.

Proof.

BASIS ($P(1)$):

$$LHS = \sum_{i=1}^{1} i$$

Induction (contd.)

Example

Show that the sum of the first n integers is $\frac{n\cdot(n+1)}{2}$. Formally, $\sum_{i=1}^{n} i = \frac{n\cdot(n+1)}{2}$.

Proof.

BASIS ($P(1)$):

$$\begin{aligned} LHS &= \sum_{i=1}^{1} i \\ &= 1 \end{aligned}$$

Induction (contd.)

Example

Show that the sum of the first n integers is $\frac{n\cdot(n+1)}{2}$. Formally, $\sum_{i=1}^{n} i = \frac{n\cdot(n+1)}{2}$.

Proof.

BASIS ($P(1)$):

$$\begin{aligned} LHS &= \sum_{i=1}^{1} i \\ &= 1 \\ RHS &= \frac{1\cdot(1+1)}{2} \end{aligned}$$

Induction (contd.)

Example

Show that the sum of the first n integers is $\frac{n\cdot(n+1)}{2}$. Formally, $\sum_{i=1}^{n} i = \frac{n\cdot(n+1)}{2}$.

Proof.

BASIS ($P(1)$):

$$\begin{aligned} LHS &= \sum_{i=1}^{1} i \\ &= 1 \\ RHS &= \frac{1\cdot(1+1)}{2} \\ &= \frac{1\cdot(2)}{2} \end{aligned}$$

Induction (contd.)

Example

Show that the sum of the first n integers is $\frac{n\cdot(n+1)}{2}$. Formally, $\sum_{i=1}^{n} i = \frac{n\cdot(n+1)}{2}$.

Proof.

BASIS ($P(1)$):

$$\begin{aligned} LHS &= \sum_{i=1}^{1} i \\ &= 1 \\ RHS &= \frac{1\cdot(1+1)}{2} \\ &= \frac{1\cdot(2)}{2} \\ &= \frac{2}{2} \end{aligned}$$

Induction (contd.)

Example

Show that the sum of the first n integers is $\frac{n\cdot(n+1)}{2}$. Formally, $\sum_{i=1}^{n} i = \frac{n\cdot(n+1)}{2}$.

Proof.

BASIS ($P(1)$):

$$\begin{aligned} LHS &= \sum_{i=1}^{1} i \\ &= 1 \\ RHS &= \frac{1\cdot(1+1)}{2} \\ &= \frac{1\cdot(2)}{2} \\ &= \frac{2}{2} \\ &= 1 \end{aligned}$$

Induction (contd.)

Example

Show that the sum of the first n integers is $\frac{n\cdot(n+1)}{2}$. Formally, $\sum_{i=1}^{n} i = \frac{n\cdot(n+1)}{2}$.

Proof.

BASIS ($P(1)$):

$$\begin{aligned} LHS &= \sum_{i=1}^{1} i \\ &= 1 \\ RHS &= \frac{1\cdot(1+1)}{2} \\ &= \frac{1\cdot(2)}{2} \\ &= \frac{2}{2} \\ &= 1 \end{aligned}$$

Thus, $LHS = RHS$ and $P(1)$ is true. □

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i = \frac{k \cdot (k+1)}{2}.$$

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i = \frac{k \cdot (k+1)}{2}.$$

We need to show that $P(k+1)$ is true,

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i = \frac{k \cdot (k+1)}{2}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i = \frac{(k+1)\cdot(k+2)}{2}$.

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i = \frac{k \cdot (k+1)}{2}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i = \frac{(k+1)\cdot(k+2)}{2}$.

$$LHS = \sum_{i=1}^{k+1} i$$

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i = \frac{k \cdot (k+1)}{2}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i = \frac{(k+1)\cdot(k+2)}{2}$.

$$\begin{aligned} LHS &= \sum_{i=1}^{k+1} i \\ &= 1 + 2 + 3 + \ldots + k + (k+1) \end{aligned}$$

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i = \frac{k \cdot (k+1)}{2}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i = \frac{(k+1)\cdot(k+2)}{2}$.

$$\begin{aligned} LHS &= \sum_{i=1}^{k+1} i \\ &= 1 + 2 + 3 + \ldots + k + (k+1) \\ &= (1 + 2 + 3 + \ldots + k) + (k+1) \end{aligned}$$

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i = \frac{k \cdot (k+1)}{2}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i = \frac{(k+1)\cdot(k+2)}{2}$.

$$\begin{aligned}
LHS &= \sum_{i=1}^{k+1} i \\
&= 1 + 2 + 3 + \ldots + k + (k+1) \\
&= (1 + 2 + 3 + \ldots + k) + (k+1) \\
&= \frac{k \cdot (k+1)}{2} + (k+1), \quad \text{using the inductive hypothesis}
\end{aligned}$$

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i = \frac{k \cdot (k+1)}{2}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i = \frac{(k+1)\cdot(k+2)}{2}$.

$$\begin{aligned}
LHS &= \sum_{i=1}^{k+1} i \\
&= 1+2+3+\ldots+k+(k+1) \\
&= (1+2+3+\ldots+k)+(k+1) \\
&= \frac{k \cdot (k+1)}{2} + (k+1), \quad \text{using the inductive hypothesis} \\
&= \frac{k+1}{2}(k+2)
\end{aligned}$$

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i = \frac{k \cdot (k+1)}{2}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i = \frac{(k+1)\cdot(k+2)}{2}$.

$$\begin{aligned}
LHS &= \sum_{i=1}^{k+1} i \\
&= 1 + 2 + 3 + \ldots + k + (k+1) \\
&= (1 + 2 + 3 + \ldots + k) + (k+1) \\
&= \frac{k \cdot (k+1)}{2} + (k+1), \quad \text{using the inductive hypothesis} \\
&= \frac{k+1}{2}(k+2) \\
&= \frac{(k+1) \cdot (k+2)}{2}
\end{aligned}$$

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i = \frac{k \cdot (k+1)}{2}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i = \frac{(k+1) \cdot (k+2)}{2}$.

$$\begin{aligned}
LHS &= \sum_{i=1}^{k+1} i \\
&= 1 + 2 + 3 + \ldots + k + (k+1) \\
&= (1 + 2 + 3 + \ldots + k) + (k+1) \\
&= \frac{k \cdot (k+1)}{2} + (k+1), \quad \text{using the inductive hypothesis} \\
&= \frac{k+1}{2}(k+2) \\
&= \frac{(k+1) \cdot (k+2)}{2} \\
&= RHS.
\end{aligned}$$

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i = \frac{k \cdot (k+1)}{2}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i = \frac{(k+1)\cdot(k+2)}{2}$.

$$\begin{aligned}
LHS &= \sum_{i=1}^{k+1} i \\
&= 1 + 2 + 3 + \ldots + k + (k+1) \\
&= (1 + 2 + 3 + \ldots + k) + (k+1) \\
&= \frac{k \cdot (k+1)}{2} + (k+1), \quad \text{using the inductive hypothesis} \\
&= \frac{k+1}{2}(k+2) \\
&= \frac{(k+1)\cdot(k+2)}{2} \\
&= RHS.
\end{aligned}$$

Since, *LHS*=*RHS*, we have shown that $P(k) \rightarrow P(k+1)$.

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i = \frac{k \cdot (k+1)}{2}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i = \frac{(k+1) \cdot (k+2)}{2}$.

$$\begin{aligned}
LHS &= \sum_{i=1}^{k+1} i \\
&= 1 + 2 + 3 + \ldots + k + (k+1) \\
&= (1 + 2 + 3 + \ldots + k) + (k+1) \\
&= \frac{k \cdot (k+1)}{2} + (k+1), \quad \text{using the inductive hypothesis} \\
&= \frac{k+1}{2}(k+2) \\
&= \frac{(k+1) \cdot (k+2)}{2} \\
&= RHS.
\end{aligned}$$

Since, *LHS=RHS*, we have shown that $P(k) \rightarrow P(k+1)$.

Applying the first principle of mathematical induction, we conclude that the conjecture is true. □

Principles

Main Ideas

Principles

Main Ideas

(i) Mathematicize the conjecture.

Principles

Main Ideas

(i) Mathematicize the conjecture.

(ii) Prove the basis (usually $P(1)$ and usually easy.)

Principles

Main Ideas

(i) Mathematicize the conjecture.

(ii) Prove the basis (usually $P(1)$ and usually easy.)

(iii) Assume $P(k)$.

Principles

Main Ideas

(i) Mathematicize the conjecture.

(ii) Prove the basis (usually $P(1)$ and usually easy.)

(iii) Assume $P(k)$.

(iv) Show $P(k+1)$.

Principles

Main Ideas

(i) Mathematicize the conjecture.
(ii) Prove the basis (usually $P(1)$ and usually easy.)
(iii) Assume $P(k)$.
(iv) Show $P(k+1)$. (The hard part.

Principles

Main Ideas

(i) Mathematicize the conjecture.
(ii) Prove the basis (usually $P(1)$ and usually easy.)
(iii) Assume $P(k)$.
(iv) Show $P(k+1)$. (The hard part. Use mathematical manipulation.)

Another Induction Example

Example

Show that the sum of the squares of the first n integers is $\frac{n\cdot(n+1)\cdot(2n+1)}{6}$,

Another Induction Example

Example

Show that the sum of the squares of the first n integers is $\frac{n\cdot(n+1)\cdot(2n+1)}{6}$, i.e., show that $\sum_{i=1}^{n} i^2 = \frac{n\cdot(n+1)\cdot(2n+1)}{6}$.

Another Induction Example

Example

Show that the sum of the squares of the first n integers is $\frac{n\cdot(n+1)\cdot(2n+1)}{6}$, i.e., show that $\sum_{i=1}^{n} i^2 = \frac{n\cdot(n+1)\cdot(2n+1)}{6}$.

Proof.

BASIS ($P(1)$):

$$LHS = \sum_{i=1}^{1} i^2$$

Another Induction Example

Example

Show that the sum of the squares of the first n integers is $\frac{n\cdot(n+1)\cdot(2n+1)}{6}$, i.e., show that $\sum_{i=1}^{n} i^2 = \frac{n\cdot(n+1)\cdot(2n+1)}{6}$.

Proof.

BASIS ($P(1)$):

$$\begin{aligned} LHS &= \sum_{i=1}^{1} i^2 \\ &= 1 \end{aligned}$$

Another Induction Example

Example

Show that the sum of the squares of the first n integers is $\frac{n\cdot(n+1)\cdot(2n+1)}{6}$, i.e., show that $\sum_{i=1}^{n} i^2 = \frac{n\cdot(n+1)\cdot(2n+1)}{6}$.

Proof.

BASIS ($P(1)$):

$$\begin{aligned} LHS &= \sum_{i=1}^{1} i^2 \\ &= 1 \\ RHS &= \frac{1\cdot(1+1)\cdot(2\cdot 1+1)}{6} \end{aligned}$$

Another Induction Example

Example

Show that the sum of the squares of the first n integers is $\frac{n\cdot(n+1)\cdot(2n+1)}{6}$, i.e., show that $\sum_{i=1}^{n} i^2 = \frac{n\cdot(n+1)\cdot(2n+1)}{6}$.

Proof.

BASIS ($P(1)$):

$$\begin{aligned} LHS &= \sum_{i=1}^{1} i^2 \\ &= 1 \\ RHS &= \frac{1 \cdot (1+1) \cdot (2 \cdot 1 + 1)}{6} \\ &= \frac{1 \cdot (2) \cdot (3)}{6} \end{aligned}$$

Another Induction Example

Example

Show that the sum of the squares of the first n integers is $\frac{n\cdot(n+1)\cdot(2n+1)}{6}$, i.e., show that $\sum_{i=1}^{n} i^2 = \frac{n\cdot(n+1)\cdot(2n+1)}{6}$.

Proof.

BASIS ($P(1)$):

$$
\begin{aligned}
LHS &= \sum_{i=1}^{1} i^2 \\
&= 1 \\
RHS &= \frac{1 \cdot (1+1) \cdot (2 \cdot 1 + 1)}{6} \\
&= \frac{1 \cdot (2) \cdot (3)}{6} \\
&= \frac{6}{6}
\end{aligned}
$$

Another Induction Example

Example

Show that the sum of the squares of the first n integers is $\frac{n\cdot(n+1)\cdot(2n+1)}{6}$, i.e., show that $\sum_{i=1}^{n} i^2 = \frac{n\cdot(n+1)\cdot(2n+1)}{6}$.

Proof.

BASIS ($P(1)$):

$$\begin{aligned} LHS &= \sum_{i=1}^{1} i^2 \\ &= 1 \\ RHS &= \frac{1\cdot(1+1)\cdot(2\cdot 1+1)}{6} \\ &= \frac{1\cdot(2)\cdot(3)}{6} \\ &= \frac{6}{6} \\ &= 1 \end{aligned}$$

Another Induction Example

Example

Show that the sum of the squares of the first n integers is $\frac{n\cdot(n+1)\cdot(2n+1)}{6}$, i.e., show that $\sum_{i=1}^{n} i^2 = \frac{n\cdot(n+1)\cdot(2n+1)}{6}$.

Proof.

BASIS ($P(1)$):

$$\begin{aligned}
LHS &= \sum_{i=1}^{1} i^2 \\
&= 1 \\
RHS &= \frac{1 \cdot (1+1) \cdot (2 \cdot 1 + 1)}{6} \\
&= \frac{1 \cdot (2) \cdot (3)}{6} \\
&= \frac{6}{6} \\
&= 1
\end{aligned}$$

Thus, $LHS = RHS$ and $P(1)$ is true. □

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i^2 = \frac{k \cdot (k+1) \cdot (2k+1)}{6}.$$

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i^2 = \frac{k \cdot (k+1) \cdot (2k+1)}{6}.$$

We need to show that $P(k+1)$ is true,

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i^2 = \frac{k \cdot (k+1) \cdot (2k+1)}{6}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i^2 = \frac{(k+1)\cdot(k+2)\cdot(2\cdot(k+1)+1)}{6}$.

$$LHS = \sum_{i=1}^{k+1} i^2$$

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i^2 = \frac{k \cdot (k+1) \cdot (2k+1)}{6}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i^2 = \frac{(k+1)\cdot(k+2)\cdot(2\cdot(k+1)+1)}{6}$.

$$\begin{aligned} LHS &= \sum_{i=1}^{k+1} i^2 \\ &= 1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2 \end{aligned}$$

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i^2 = \frac{k \cdot (k+1) \cdot (2k+1)}{6}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i^2 = \frac{(k+1)\cdot(k+2)\cdot(2\cdot(k+1)+1)}{6}$.

$$\begin{aligned} LHS &= \sum_{i=1}^{k+1} i^2 \\ &= 1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2 \\ &= (1^2 + 2^2 + 3^2 + \ldots + k^2) + (k+1)^2 \end{aligned}$$

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i^2 = \frac{k \cdot (k+1) \cdot (2k+1)}{6}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i^2 = \frac{(k+1)\cdot(k+2)\cdot(2\cdot(k+1)+1)}{6}$.

$$\begin{aligned} LHS &= \sum_{i=1}^{k+1} i^2 \\ &= 1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2 \\ &= (1^2 + 2^2 + 3^2 + \ldots + k^2) + (k+1)^2 \\ &= \frac{k \cdot (k+1) \cdot (2k+1)}{6} + (k+1)^2, \quad \text{using the inductive hypothesis} \end{aligned}$$

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i^2 = \frac{k \cdot (k+1) \cdot (2k+1)}{6}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i^2 = \frac{(k+1)\cdot(k+2)\cdot(2\cdot(k+1)+1)}{6}$.

$$\begin{aligned} LHS &= \sum_{i=1}^{k+1} i^2 \\ &= 1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2 \\ &= (1^2 + 2^2 + 3^2 + \ldots + k^2) + (k+1)^2 \\ &= \frac{k \cdot (k+1) \cdot (2k+1)}{6} + (k+1)^2, \quad \text{using the inductive hypothesis} \\ &= \frac{k+1}{6}(k \cdot (2k+1) + 6 \cdot (k+1)) \end{aligned}$$

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i^2 = \frac{k \cdot (k+1) \cdot (2k+1)}{6}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i^2 = \frac{(k+1)\cdot(k+2)\cdot(2\cdot(k+1)+1)}{6}$.

$$\begin{aligned}
LHS &= \sum_{i=1}^{k+1} i^2 \\
&= 1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2 \\
&= (1^2 + 2^2 + 3^2 + \ldots + k^2) + (k+1)^2 \\
&= \frac{k \cdot (k+1) \cdot (2k+1)}{6} + (k+1)^2, \quad \text{using the inductive hypothesis} \\
&= \frac{k+1}{6}(k \cdot (2k+1) + 6 \cdot (k+1)) \\
&= \frac{k+1}{6}(2k^2 + k + 6k + 6)
\end{aligned}$$

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i^2 = \frac{k \cdot (k+1) \cdot (2k+1)}{6}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i^2 = \frac{(k+1)\cdot(k+2)\cdot(2\cdot(k+1)+1)}{6}$.

$$\begin{aligned}
LHS &= \sum_{i=1}^{k+1} i^2 \\
&= 1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2 \\
&= (1^2 + 2^2 + 3^2 + \ldots + k^2) + (k+1)^2 \\
&= \frac{k \cdot (k+1) \cdot (2k+1)}{6} + (k+1)^2, \quad \text{using the inductive hypothesis} \\
&= \frac{k+1}{6}(k \cdot (2k+1) + 6 \cdot (k+1)) \\
&= \frac{k+1}{6}(2k^2 + k + 6k + 6) \\
&= \frac{k+1}{6}(2k^2 + 7k + 6)
\end{aligned}$$

Induction example (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k} i^2 = \frac{k \cdot (k+1) \cdot (2k+1)}{6}.$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1} i^2 = \frac{(k+1)\cdot(k+2)\cdot(2\cdot(k+1)+1)}{6}$.

$$\begin{aligned}
LHS &= \sum_{i=1}^{k+1} i^2 \\
&= 1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2 \\
&= (1^2 + 2^2 + 3^2 + \ldots + k^2) + (k+1)^2 \\
&= \frac{k \cdot (k+1) \cdot (2k+1)}{6} + (k+1)^2, \quad \text{using the inductive hypothesis} \\
&= \frac{k+1}{6}(k \cdot (2k+1) + 6 \cdot (k+1)) \\
&= \frac{k+1}{6}(2k^2 + k + 6k + 6) \\
&= \frac{k+1}{6}(2k^2 + 7k + 6)
\end{aligned}$$

□

Induction proof (contd.)

Proof.

$$= \quad \frac{k+1}{6}(2k^2 + 4k + 3k + 6)$$

Induction proof (contd.)

Proof.

$$= \frac{k+1}{6}(2k^2 + 4k + 3k + 6)$$

$$= \frac{k+1}{6}(2k \cdot (k+2) + 3 \cdot (k+2))$$

Induction proof (contd.)

Proof.

$$
\begin{aligned}
&= \frac{k+1}{6}(2k^2 + 4k + 3k + 6) \\
&= \frac{k+1}{6}(2k \cdot (k+2) + 3 \cdot (k+2)) \\
&= \frac{k+1}{6}(2k+3) \cdot (k+2))
\end{aligned}
$$

Induction proof (contd.)

Proof.

$$
\begin{aligned}
&= \frac{k+1}{6}(2k^2 + 4k + 3k + 6) \\
&= \frac{k+1}{6}(2k \cdot (k+2) + 3 \cdot (k+2)) \\
&= \frac{k+1}{6}(2k+3) \cdot (k+2)) \\
&= \frac{(k+1) \cdot (k+2) \cdot (2 \cdot (k+1) + 1)}{6}
\end{aligned}
$$

Induction proof (contd.)

Proof.

$$
\begin{aligned}
&= \frac{k+1}{6}(2k^2 + 4k + 3k + 6) \\
&= \frac{k+1}{6}(2k \cdot (k+2) + 3 \cdot (k+2)) \\
&= \frac{k+1}{6}(2k+3) \cdot (k+2)) \\
&= \frac{(k+1) \cdot (k+2) \cdot (2 \cdot (k+1) + 1)}{6} \\
&= RHS.
\end{aligned}
$$

Induction proof (contd.)

Proof.

$$
\begin{aligned}
&= \frac{k+1}{6}(2k^2 + 4k + 3k + 6) \\
&= \frac{k+1}{6}(2k \cdot (k+2) + 3 \cdot (k+2)) \\
&= \frac{k+1}{6}(2k+3) \cdot (k+2)) \\
&= \frac{(k+1) \cdot (k+2) \cdot (2 \cdot (k+1) + 1)}{6} \\
&= RHS.
\end{aligned}
$$

Since, *LHS*=*RHS*, we have shown that $P(k) \rightarrow P(k+1)$.

Induction proof (contd.)

Proof.

$$\begin{aligned}
&= \frac{k+1}{6}(2k^2 + 4k + 3k + 6) \\
&= \frac{k+1}{6}(2k \cdot (k+2) + 3 \cdot (k+2)) \\
&= \frac{k+1}{6}(2k+3) \cdot (k+2)) \\
&= \frac{(k+1) \cdot (k+2) \cdot (2 \cdot (k+1) + 1)}{6} \\
&= RHS.
\end{aligned}$$

Since, *LHS*=*RHS*, we have shown that $P(k) \rightarrow P(k+1)$.

Applying the first principle of mathematical induction, we conclude that the conjecture is true. □

Induction Example

Example

Show that the sum of the first n odd integers is n^2, i.e., show that $\sum_{i=1}^{n}(2i-1) = n^2$.

Induction Example

Example

Show that the sum of the first n odd integers is n^2, i.e., show that $\sum_{i=1}^{n}(2i-1) = n^2$.

Proof.

BASIS ($P(1)$):

$$LHS = \sum_{i=1}^{1}(2i-1)$$

Induction Example

Example

Show that the sum of the first n odd integers is n^2, i.e., show that $\sum_{i=1}^{n}(2i - 1) = n^2$.

Proof.

BASIS ($P(1)$):

$$\begin{aligned} LHS &= \sum_{i=1}^{1}(2i - 1) \\ &= 2 \cdot 1 - 1 \end{aligned}$$

Induction Example

Example

Show that the sum of the first n odd integers is n^2, i.e., show that $\sum_{i=1}^{n}(2i-1) = n^2$.

Proof.

Basis ($P(1)$):

$$\begin{aligned} LHS &= \sum_{i=1}^{1}(2i-1) \\ &= 2 \cdot 1 - 1 \\ &= 1 \end{aligned}$$

Induction Example

Example

Show that the sum of the first n odd integers is n^2, i.e., show that $\sum_{i=1}^{n}(2i-1) = n^2$.

Proof.

BASIS ($P(1)$):

$$\begin{aligned} LHS &= \sum_{i=1}^{1}(2i-1) \\ &= 2\cdot 1 - 1 \\ &= 1 \\ RHS &= 1^2 \end{aligned}$$

Induction Example

Example

Show that the sum of the first n odd integers is n^2, i.e., show that $\sum_{i=1}^{n}(2i-1) = n^2$.

Proof.

BASIS ($P(1)$):

$$\begin{aligned} LHS &= \sum_{i=1}^{1}(2i-1) \\ &= 2 \cdot 1 - 1 \\ &= 1 \\ RHS &= 1^2 \\ &= 1 \end{aligned}$$

Induction Example

Example

Show that the sum of the first n odd integers is n^2, i.e., show that $\sum_{i=1}^{n}(2i-1) = n^2$.

Proof.

BASIS ($P(1)$):

$$\begin{aligned} LHS &= \sum_{i=1}^{1}(2i-1) \\ &= 2 \cdot 1 - 1 \\ &= 1 \\ RHS &= 1^2 \\ &= 1 \end{aligned}$$

Thus, $LHS = RHS$ and $P(1)$ is true. □

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k}(2i - 1) = k^2$$

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k}(2i - 1) = k^2$$

We need to show that $P(k + 1)$ is true,

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k}(2i - 1) = k^2$$

We need to show that $P(k + 1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1}(2i - 1) = (k + 1)^2$.

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k}(2i - 1) = k^2$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1}(2i - 1) = (k+1)^2$.

$$LHS = \sum_{i=1}^{k+1}(2i - 1)$$

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k}(2i - 1) = k^2$$

We need to show that $P(k + 1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1}(2i - 1) = (k + 1)^2$.

$$\begin{aligned} LHS &= \sum_{i=1}^{k+1}(2i - 1) \\ &= 1 + 3 + 5 + \ldots (2k - 1) + (2(k + 1) - 1) \end{aligned}$$

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k}(2i-1) = k^2$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1}(2i-1) = (k+1)^2$.

$$\begin{aligned} LHS &= \sum_{i=1}^{k+1}(2i-1) \\ &= 1+3+5+\ldots(2k-1)+(2(k+1)-1) \\ &= (1+3+5+\ldots(2k-1))+(2k+1) \end{aligned}$$

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k}(2i - 1) = k^2$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1}(2i - 1) = (k+1)^2$.

$$\begin{aligned} LHS &= \sum_{i=1}^{k+1}(2i - 1) \\ &= 1 + 3 + 5 + \ldots (2k - 1) + (2(k+1) - 1) \\ &= (1 + 3 + 5 + \ldots (2k - 1)) + (2k + 1) \\ &= k^2 + (2k + 1), \quad \text{using the inductive hypothesis} \end{aligned}$$

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k}(2i-1) = k^2$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1}(2i-1) = (k+1)^2$.

$$\begin{aligned}
LHS &= \sum_{i=1}^{k+1}(2i-1) \\
&= 1+3+5+\ldots(2k-1)+(2(k+1)-1) \\
&= (1+3+5+\ldots(2k-1))+(2k+1) \\
&= k^2+(2k+1), \quad \text{using the inductive hypothesis} \\
&= (k+1)^2
\end{aligned}$$

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k}(2i-1) = k^2$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1}(2i-1) = (k+1)^2$.

$$\begin{aligned}
LHS &= \sum_{i=1}^{k+1}(2i-1) \\
&= 1+3+5+\ldots(2k-1)+(2(k+1)-1) \\
&= (1+3+5+\ldots(2k-1))+(2k+1) \\
&= k^2+(2k+1), \quad \text{using the inductive hypothesis} \\
&= (k+1)^2 \\
&= RHS
\end{aligned}$$

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k}(2i - 1) = k^2$$

We need to show that $P(k + 1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1}(2i - 1) = (k + 1)^2$.

$$\begin{aligned}
LHS &= \sum_{i=1}^{k+1}(2i - 1) \\
&= 1 + 3 + 5 + \ldots (2k - 1) + (2(k + 1) - 1) \\
&= (1 + 3 + 5 + \ldots (2k - 1)) + (2k + 1) \\
&= k^2 + (2k + 1), \quad \text{using the inductive hypothesis} \\
&= (k + 1)^2 \\
&= RHS
\end{aligned}$$

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k}(2i-1) = k^2$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1}(2i-1) = (k+1)^2$.

$$\begin{aligned}
LHS &= \sum_{i=1}^{k+1}(2i-1) \\
&= 1+3+5+\ldots(2k-1)+(2(k+1)-1) \\
&= (1+3+5+\ldots(2k-1))+(2k+1) \\
&= k^2+(2k+1), \quad \text{using the inductive hypothesis} \\
&= (k+1)^2 \\
&= RHS
\end{aligned}$$

Since $LHS = RHS$, we have shown that $P(k) \rightarrow P(k+1)$.

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that

$$\sum_{i=1}^{k}(2i-1) = k^2$$

We need to show that $P(k+1)$ is true, i.e., we need to show that $\sum_{i=1}^{k+1}(2i-1) = (k+1)^2$.

$$\begin{aligned}
LHS &= \sum_{i=1}^{k+1}(2i-1) \\
&= 1+3+5+\ldots(2k-1)+(2(k+1)-1) \\
&= (1+3+5+\ldots(2k-1))+(2k+1) \\
&= k^2+(2k+1), \quad \text{using the inductive hypothesis} \\
&= (k+1)^2 \\
&= RHS
\end{aligned}$$

Since $LHS = RHS$, we have shown that $P(k) \to P(k+1)$. Applying the first principle of mathematical induction, we conclude that the conjecture is true. □

One Final Example

Example

Show that $7^n - 5^n$ is always an even number for $n \geq 0$, i.e., show that $2 \mid (7^n - 5^n)$, $\forall n \geq 0$.

One Final Example

Example

Show that $7^n - 5^n$ is always an even number for $n \geq 0$, i.e., show that $2 \mid (7^n - 5^n)$, $\forall n \geq 0$.

Proof.

BASIS ($P(0)$):

$$LHS = 7^0 - 5^0$$

One Final Example

Example

Show that $7^n - 5^n$ is always an even number for $n \geq 0$, i.e., show that $2 \mid (7^n - 5^n)$, $\forall n \geq 0$.

Proof.

BASIS ($P(0)$):

$$\begin{aligned} LHS &= 7^0 - 5^0 \\ &= 1 - 1 \end{aligned}$$

One Final Example

Example

Show that $7^n - 5^n$ is always an even number for $n \geq 0$, i.e., show that $2 \mid (7^n - 5^n)$, $\forall n \geq 0$.

Proof.

BASIS ($P(0)$):

$$\begin{aligned} LHS &= 7^0 - 5^0 \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

One Final Example

Example

Show that $7^n - 5^n$ is always an even number for $n \geq 0$, i.e., show that $2 \mid (7^n - 5^n)$, $\forall n \geq 0$.

Proof.

BASIS ($P(0)$):

$$\begin{aligned} LHS &= 7^0 - 5^0 \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

Since the LHS is even, we have proven the basis $P(0)$. □

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that $(7^k - 5^k)$ is divisible by 2 for some k.

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that $(7^k - 5^k)$ is divisible by 2 for some k. It follows that $(7^k - 5^k) = 2m$, for some integer m.

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that $(7^k - 5^k)$ is divisible by 2 for some k. It follows that $(7^k - 5^k) = 2m$, for some integer m. We need to show that $P(k+1)$ is true, i.e., $(7^{k+1} - 5^{k+1})$ is divisible by 2.

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that $(7^k - 5^k)$ is divisible by 2 for some k. It follows that $(7^k - 5^k) = 2m$, for some integer m. We need to show that $P(k+1)$ is true, i.e., $(7^{k+1} - 5^{k+1})$ is divisible by 2. Observe that,

$$7^{k+1} - 5^{k+1} = 7 \cdot 7^k - 5 \cdot 5^k$$

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that $(7^k - 5^k)$ is divisible by 2 for some k. It follows that $(7^k - 5^k) = 2m$, for some integer m. We need to show that $P(k+1)$ is true, i.e., $(7^{k+1} - 5^{k+1})$ is divisible by 2. Observe that,

$$\begin{aligned} 7^{k+1} - 5^{k+1} &= 7 \cdot 7^k - 5 \cdot 5^k \\ &= 7 \cdot (2m + 5^k) - 5 \cdot 5^k, \text{ using the inductive hypothesis} \end{aligned}$$

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that $(7^k - 5^k)$ is divisible by 2 for some k. It follows that $(7^k - 5^k) = 2m$, for some integer m. We need to show that $P(k+1)$ is true, i.e., $(7^{k+1} - 5^{k+1})$ is divisible by 2. Observe that,

$$\begin{aligned} 7^{k+1} - 5^{k+1} &= 7 \cdot 7^k - 5 \cdot 5^k \\ &= 7 \cdot (2m + 5^k) - 5 \cdot 5^k, \text{ using the inductive hypothesis} \\ &= 14m + 7 \cdot 5^k - 5 \cdot 5^k \end{aligned}$$

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that $(7^k - 5^k)$ is divisible by 2 for some k. It follows that $(7^k - 5^k) = 2m$, for some integer m. We need to show that $P(k+1)$ is true, i.e., $(7^{k+1} - 5^{k+1})$ is divisible by 2. Observe that,

$$\begin{aligned}
7^{k+1} - 5^{k+1} &= 7 \cdot 7^k - 5 \cdot 5^k \\
&= 7 \cdot (2m + 5^k) - 5 \cdot 5^k, \text{ using the inductive hypothesis} \\
&= 14m + 7 \cdot 5^k - 5 \cdot 5^k \\
&= 14m + 5^k \cdot (7 - 5)
\end{aligned}$$

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that $(7^k - 5^k)$ is divisible by 2 for some k. It follows that $(7^k - 5^k) = 2m$, for some integer m. We need to show that $P(k+1)$ is true, i.e., $(7^{k+1} - 5^{k+1})$ is divisible by 2. Observe that,

$$\begin{aligned}
7^{k+1} - 5^{k+1} &= 7 \cdot 7^k - 5 \cdot 5^k \\
&= 7 \cdot (2m + 5^k) - 5 \cdot 5^k, \text{ using the inductive hypothesis} \\
&= 14m + 7 \cdot 5^k - 5 \cdot 5^k \\
&= 14m + 5^k \cdot (7 - 5) \\
&= 14m + 2 \cdot 5^k
\end{aligned}$$

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that $(7^k - 5^k)$ is divisible by 2 for some k. It follows that $(7^k - 5^k) = 2m$, for some integer m. We need to show that $P(k+1)$ is true, i.e., $(7^{k+1} - 5^{k+1})$ is divisible by 2. Observe that,

$$\begin{aligned}
7^{k+1} - 5^{k+1} &= 7 \cdot 7^k - 5 \cdot 5^k \\
&= 7 \cdot (2m + 5^k) - 5 \cdot 5^k, \quad \text{using the inductive hypothesis} \\
&= 14m + 7 \cdot 5^k - 5 \cdot 5^k \\
&= 14m + 5^k \cdot (7 - 5) \\
&= 14m + 2 \cdot 5^k \\
&= 2 \cdot (7m + 5^k)
\end{aligned}$$

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that $(7^k - 5^k)$ is divisible by 2 for some k. It follows that $(7^k - 5^k) = 2m$, for some integer m. We need to show that $P(k+1)$ is true, i.e., $(7^{k+1} - 5^{k+1})$ is divisible by 2. Observe that,

$$\begin{aligned}
7^{k+1} - 5^{k+1} &= 7 \cdot 7^k - 5 \cdot 5^k \\
&= 7 \cdot (2m + 5^k) - 5 \cdot 5^k, \text{ using the inductive hypothesis} \\
&= 14m + 7 \cdot 5^k - 5 \cdot 5^k \\
&= 14m + 5^k \cdot (7 - 5) \\
&= 14m + 2 \cdot 5^k \\
&= 2 \cdot (7m + 5^k) \\
&= \text{some even number!}
\end{aligned}$$

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that $(7^k - 5^k)$ is divisible by 2 for some k. It follows that $(7^k - 5^k) = 2m$, for some integer m. We need to show that $P(k+1)$ is true, i.e., $(7^{k+1} - 5^{k+1})$ is divisible by 2. Observe that,

$$
\begin{aligned}
7^{k+1} - 5^{k+1} &= 7 \cdot 7^k - 5 \cdot 5^k \\
&= 7 \cdot (2m + 5^k) - 5 \cdot 5^k, \text{ using the inductive hypothesis} \\
&= 14m + 7 \cdot 5^k - 5 \cdot 5^k \\
&= 14m + 5^k \cdot (7 - 5) \\
&= 14m + 2 \cdot 5^k \\
&= 2 \cdot (7m + 5^k) \\
&= \text{some even number!}
\end{aligned}
$$

We have thus shown that $P(k) \rightarrow P(k+1)$.

Proof (contd.)

Proof.

Let us assume that $P(k)$ is true, i.e., assume that $(7^k - 5^k)$ is divisible by 2 for some k. It follows that $(7^k - 5^k) = 2m$, for some integer m. We need to show that $P(k+1)$ is true, i.e., $(7^{k+1} - 5^{k+1})$ is divisible by 2. Observe that,

$$
\begin{aligned}
7^{k+1} - 5^{k+1} &= 7 \cdot 7^k - 5 \cdot 5^k \\
&= 7 \cdot (2m + 5^k) - 5 \cdot 5^k, \text{ using the inductive hypothesis} \\
&= 14m + 7 \cdot 5^k - 5 \cdot 5^k \\
&= 14m + 5^k \cdot (7 - 5) \\
&= 14m + 2 \cdot 5^k \\
&= 2 \cdot (7m + 5^k) \\
&= \text{some even number!}
\end{aligned}
$$

We have thus shown that $P(k) \rightarrow P(k+1)$. Applying the first principle of mathematical induction, we conclude that the conjecture is true. □

Second Principle of Induction

Note

Also called Strong Induction. Is necessary, when the first principle does not help us.

Second Principle of Induction

Note

Also called Strong Induction. Is necessary, when the first principle does not help us.

Principle

Assume that the domain is the set of integers.

Second Principle of Induction

Note

Also called Strong Induction. Is necessary, when the first principle does not help us.

Principle

Assume that the domain is the set of integers.

(i) $P(1)$ is **true**.

Second Principle of Induction

Note

Also called Strong Induction. Is necessary, when the first principle does not help us.

Principle

Assume that the domain is the set of integers.

(i) $P(1)$ is **true**.

(ii) $(\forall r)[P(r)$ true for all r, $1 \leq r \leq k \rightarrow P(k+1)]$

Second Principle of Induction

Note

Also called Strong Induction. Is necessary, when the first principle does not help us.

Principle

Assume that the domain is the set of integers.

(i) $P(1)$ is **true**.
(ii) $(\forall r)[P(r)$ true for all r, $1 \leq r \leq k \rightarrow P(k+1)]$

$P(n)$ is **true** for all n.

Example

Show that every number greater than or equal to 8 can be expressed in the form $5a + 3b$, for suitably chosen a and b.

Example

Show that every number greater than or equal to 8 can be expressed in the form $5a + 3b$, for suitably chosen a and b.

Proof.

The conjecture is clearly true for 8, 9 and 10.

Example

Show that every number greater than or equal to 8 can be expressed in the form $5a + 3b$, for suitably chosen a and b.

Proof.

The conjecture is clearly true for 8, 9 and 10. Assume that the conjecture holds for all r, $8 \leq r \leq k$.

Example

Show that every number greater than or equal to 8 can be expressed in the form $5a + 3b$, for suitably chosen a and b.

Proof.

The conjecture is clearly true for 8, 9 and 10. Assume that the conjecture holds for all r, $8 \leq r \leq k$. Consider the integer $k + 1$.

Example

Show that every number greater than or equal to 8 can be expressed in the form $5a + 3b$, for suitably chosen a and b.

Proof.

The conjecture is clearly true for 8, 9 and 10. Assume that the conjecture holds for all r, $8 \leq r \leq k$. Consider the integer $k+1$. Without loss of generality, we assume that $(k+1) \geq 11$.

Example

Show that every number greater than or equal to 8 can be expressed in the form $5a + 3b$, for suitably chosen a and b.

Proof.

The conjecture is clearly true for 8, 9 and 10. Assume that the conjecture holds for all r, $8 \leq r \leq k$. Consider the integer $k+1$. Without loss of generality, we assume that $(k+1) \geq 11$. Observe that $(k+1) - 3 = k - 2$ is at least 8 and less than k.

Example

Show that every number greater than or equal to 8 can be expressed in the form $5a + 3b$, for suitably chosen a and b.

Proof.

The conjecture is clearly true for 8, 9 and 10. Assume that the conjecture holds for all r, $8 \leq r \leq k$. Consider the integer $k+1$. Without loss of generality, we assume that $(k+1) \geq 11$. Observe that $(k+1) - 3 = k - 2$ is at least 8 and less than k. As per the inductive hypothesis, $k-2$ can be expressed in the form $3a + 5b$, for suitably chosen a and b.

Example

Show that every number greater than or equal to 8 can be expressed in the form $5a + 3b$, for suitably chosen a and b.

Proof.

The conjecture is clearly true for 8, 9 and 10. Assume that the conjecture holds for all r, $8 \leq r \leq k$. Consider the integer $k + 1$. Without loss of generality, we assume that $(k + 1) \geq 11$. Observe that $(k + 1) - 3 = k - 2$ is at least 8 and less than k. As per the inductive hypothesis, $k - 2$ can be expressed in the form $3a + 5b$, for suitably chosen a and b. It follows that $(k + 1) = 3(a + 1) + 5b$, can also be so expressed.

Example

Show that every number greater than or equal to 8 can be expressed in the form $5a + 3b$, for suitably chosen a and b.

Proof.

The conjecture is clearly true for 8, 9 and 10. Assume that the conjecture holds for all r, $8 \leq r \leq k$. Consider the integer $k + 1$. Without loss of generality, we assume that $(k + 1) \geq 11$. Observe that $(k + 1) - 3 = k - 2$ is at least 8 and less than k. As per the inductive hypothesis, $k - 2$ can be expressed in the form $3a + 5b$, for suitably chosen a and b. It follows that $(k + 1) = 3(a + 1) + 5b$, can also be so expressed. Applying the second principle of mathematical induction, we conclude that the conjecture is true. □