Discrete Probability - Random Variables

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8 February, 2011

Outline

1. Recap

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Recap

Main points

Random experiment,

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Random experiment, sample spaces,

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Random experiment, sample spaces, events,

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$$P\{X = 1\} \quad =$$

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Let X denote the random variable that is defined as the sum of two fair dice. What are the values that X can take?

$$\begin{aligned} P\{X = 1\} &= 0 \\ P\{X = 2\} &= \frac{1}{36} \end{aligned}$$

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Some books refer to the probability mass function as the probability density function (pdf).

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Finally,

$$P\{X = x \mid Y = y\} = \frac{P\{X = x \text{ and } Y = y\}}{P\{Y = y\}}$$

Independence of Random Variables

Independent random variables

Two random variables X and Y are said to be independent, if

$$P\{X = x \text{ and } Y = y\} = P\{X = x\} \cdot P\{Y = y\}$$

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$$\begin{aligned} p(1) &= P\{X = 1\} = p \\ p(0) &= P\{X = 0\} = 1 - p \end{aligned}$$

where $0 \leq p \leq 1$ is the probability that the experiment results in a success.

The Binomial Random Variable

Motivation

Consider an experiment which consists of n independent Bernoulli trials, with the probability of success in each trial being p. If X is the random variable that counts the number of successes in the n trials, then X is said to be a Binomial Random Variable.

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$$\begin{aligned} E[X] &= \sum_{i=0}^{n} i \cdot p(i), \text{ by definition} \\ &= \sum_{i=0}^{n} i \cdot C(n, i) \cdot p^{i} \cdot (1-p)^{n-i} \end{aligned}$$

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Substituting $k = i - 1$, we get,

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&= n \cdot p \sum_{k=0}^{n-1} C(n-1, k) \cdot p^k \cdot (1-p)^{(n-1)-k} \\
&= n \cdot p \cdot [p + (1-p)]^{n-1}, \text{ Binomial theorem}
\end{aligned}$$

Expectation of a Binomial Random Variable (contd.)

Example

Substituting $k = i - 1$, we get,

$$\begin{aligned} E[X] &= n \cdot p \sum_{k=0}^{n-1} \frac{(n-1)!}{k! \cdot (n-k-1)!} \cdot p^k \cdot (1-p)^{n-k-1} \\ &= n \cdot p \sum_{k=0}^{n-1} \frac{(n-1)!}{k! \cdot ((n-1)-k)!} \cdot p^k \cdot (1-p)^{(n-1)-k} \\ &= n \cdot p \sum_{k=0}^{n-1} C(n-1,k) \cdot p^k \cdot (1-p)^{(n-1)-k} \\ &= n \cdot p \cdot [p + (1-p)]^{n-1}, \text{ Binomial theorem} \\ &= n \cdot p \cdot 1 \end{aligned}$$

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Let X denote a Geometric Random Variable with parameters n and p.

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$\square$

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Let X be a random variable, with the following pmf:

$$p(0) = 0.3, \ p(1) = 0.5, \ p(2) = 0.2$$

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Accordingly,

$$E[Y] = E[X^2] = 0 \cdot 0.3 + 1 \cdot 0.5 + 4 \cdot 0.2 = 1.3$$

Expectation of functions - The Direct Approach

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If X is a random variable with pmf $p()$, and $g()$ is any real-valued function, then,

$$E[g(X)] = \sum_{x:\ p(x)>0} g(x) \cdot p(x)$$

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Note that linearity of expectation holds even when the random variables are **not** *independent.*

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Note

Note that linearity of expectation holds even when the random variables are **not** *independent.*

Example

What is the expected value of the sum of the upturned faces, when two fair dice are tossed?

Expectation of product

Independent random variable

If X and Y are independent random variables (i.r.v.s), then

$$E[X \cdot Y] = E[X] \cdot E[Y]$$

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Another Application

Example

Compute the expected value of the Binomial random variable.

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Accordingly, the Binomial random variable (say X) can be expressed as:

$$X = X_1 + X_2 + \ldots X_n$$

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Definition

Basics

The expected value of a random variable does not contain any information about the "spread" of values. For instance, $E[X] = \frac{1}{2}$ for the distribution $P\{X = 0\} = P\{X = 1\} = \frac{1}{2}$ and the distribution $P\{X = \frac{1}{4}\} = P\{X = \frac{3}{4}\} = \frac{1}{2}$.

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The variance of a random variable X, with mean $E[X]$ is:

$$\begin{aligned} Var[X] &= E[(X - E[X])^2] \\ &= E[X^2] - (E[X])^2 \end{aligned}$$

Laws of Variances

Note

(i) *If a is a constant, then* $Var[aX] = a^2 Var[X]$.

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Standard Deviation

The positive square root of the variance of a random variable is called its standard deviation.

Exercises

Exercise

Compute the variances of the following random variables:

(i) *Bernoulli random variable.*
(ii) *Binomial random variable.*
(iii) *Geometric random variable.*

Computing Expectations by conditioning

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Let X and Y denote two random variables. What is $E[X \mid Y]$? $E[X \mid Y]$ is that function of the random variable Y whose value at $Y = y$ is $E[X \mid Y = y]$.

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