Design of Algorithms - Homework II

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1 Instructions

- (i) The homework is due on March 10, in class.
- (ii) Attempt as many problems as you can. You will get partial credit, as per the policy discussed in class.
- (iii) The bonus problem will add to your overall score; however, grading will be very strict on this problem.

2 Problems

- 1. Each of *n* customers gives a hat to a hat-check person at a restaurant. The hat-check person gives the hats back to the customers in a random order. What is the expected number of customers that get their own hat back?
- 2. Let $\mathbf{A}[1 \cdot n]$ represent an array of *n* distinct numbers that have been randomly permuted. If i < j and A[i] > A[j], then the pair (i, j) is called an *inversion* of \mathbf{A} . Use indicator random variables to compute the expected number of inversions of \mathbf{A} .
- 3. A miner is trapped in a mine containing three doors. The first door leads to a tunnel that will allow the miner to reach safety in two hours. The second door leads to a tunnel that will bring him back to the mine after three hours. The third door leads to a tunnel that will bring him back to the mine after five hours. Assume that the miner chooses one of the three doors uniformly and at random, whenever he is confronted with the three choices. In how much time can he expect to get to safety?
- 4. Show that

$$\sum_{k=1}^{n-1} k \cdot \log k \le \frac{1}{2} n^2 \log n - \frac{1}{8} n^2.$$

5. Show that the second smallest of n elements in an array can be found using at most $n + \lceil \log n \rceil - 2$ element to element comparisons.

BONUS: Analyze the RANDOMIZED-SELECT() algorithm discussed in class using the decision-tree based analysis to show that it runs in expected time O(n), on an array of n elements.