# Concepts in Mathematical Logic

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## 1 General Information

- (a) Instructor: K. Subramani.
- (b) Meeting Times: Monday 9:00 am 10:15 am. Location: 749 ESB.
- (c) Contact Information: 749 ESB, ksmani@csee.wvu.edu.
- (d) Office Hours: By appointment.
- (e) Textbook [Sri08].
- (f) Assessment:
  - (a) Presentations You will be required to present allotted material. The presentations will be graded on the basis of clarity, comprehensiveness and effectiveness. The presentations will be worth 80% of your grade.
  - (b) Homework Assignments You will be handed several homework assignments. These assignments constitute 20% of your grade.

A maximum of 5 bonus points will be awarded for class performance.

- (g) Grade Boundaries
  - (a) A: 80 and up
  - (b) **B**: 65 79
  - (c)  $\mathbb{C}$ : 50 64
  - (d) **D**: 45 49
  - (e) **F**: 0 44
- (h) Grading policy If you have any questions about the grading, you must contact the instructor within two days of your paper being returned.
- (i) Makeup Policy If for some reason, you are unable to attend a test or an exam, please meet me at the earliest and I will set an alternate date.
- (j) **Course Objectives** The objectives of this course are as follows:
  - (a) To reintroduce students to concepts from logic.
  - (b) To reinforce the notion of semantics from both first-order and second-order perspectives.
  - (c) To introduce students to concepts from Model theory.

- (d) To introduce students to concepts from Recursion theory.
- (k) Learning Outcomes Upon successful completion of this course, students will be:
  - (i) Able to reason using propositional logic.
  - (ii) Able to propose and prove meta-theorems in propositional and first-order logic.
  - (iii) Familiar with the completeness and compactness theorems of first-order logic.
  - (iv) Familiar with compactness theorems and complete theories in first-order logic.
  - (v) Familiar with Incompleteness theorems and Recursion theory.

## 1.1 Syntax and Semantics of First-Order Logic

First-Order Languages, Terms, Formulas and Theories of a Language, Structures of First-Order Languages, Truth in a Structure, Model of a Theory, Embeddings and Isomorphisms. This material will be covered from Chapter 1 and Chapter 2 of [Sri08].

## 1.2 Propositional Logic

Syntax, Semantics, Compactness Theorem, Proof, Metatheorems, Post Tautology Theorem. This material will be covered from Chapter 3 of [Sri08].

## 1.3 Proof and Metatheorems in First-Order Logic

Proofs, Metatheorems in First-Order Logic and Arithmetic, Consistency and Completeness. This material will be covered from Chapter 4 of [Sri08].

## 1.4 Completeness Theorem and Model Theory

Completeness Theorem, Interpretations in a Theory, Extension by Definitions, Compactness Theorem and Applications, Complete Theories, Applications in Algebra. This material will be covered from Chapter 5 of [Sri08].

#### 1.5 Recursive Functions and Arithmetization of Theories

Recursive Functions, Semirecursive predicates, Arithmetization of Theories, Decidable Theories. This material will be covered from Chapter 6 of [Sri08].

#### 1.6 Incompleteness Theorems and Recursion Theory

Representability, First Incompleteness Theorem, Arithmetical Sets, Recursive Extensions of Peano Arithmetic, Second Incompleteness Theorem.

I would like to reiterate that this is a sketch of the topics that we will be covering. For various reasons, I may choose to drop a mentioned topic or cover a new topic. In such cases, advance notice will be given.

## 2 Academic Integrity Statement

The integrity of the classes offered by any academic institution solidifies the foundation of its mission and cannot be sacrificed to expediency, ignorance, or blatant fraud. Therefore, I will enforce rigorous standards of academic integrity in all aspects and assignments of this course. For the detailed policy of West Virginia University regarding the definitions of acts considered to fall under academic dishonesty and possible ensuing sanctions, please see the Student Conduct Code

at <a href="http://www.arc.wvu.edu/admissions/integrity.html">http://www.arc.wvu.edu/admissions/integrity.html</a>. Should you have any questions about possibly improper research citations or references, or any other activity that may be interpreted as an attempt at academic dishonesty, please see me before the assignment is due to discuss the matter.

## **3 Social Justice Statement**

West Virginia University is committed to social justice. I concur with that commitment and expect to foster a nurturing learning environment, based upon open communication, mutual respect and non-discrimination. Our University does not discriminate on the basis of race, sex, age, disability, veteran status, religion, sexual orientation, color or national origin. Any suggestions to further such a positive and open environment in this class will be appreciated and given serious consideration. If you are a person with a disability and anticipate needing any type accommodation, in order to participate in this class, please advise me of the same and make appropriate arrangements with Disability Services (293 - 6700).

If you feel that you are being treated inappropriately or unfairly in any way, please feel free to bring your concerns to my attention; rest assured that doing so will not prejudice the grading process. In return, I expect you to behave professionally and ethically.

## References

[Sri08] S. M. Srivastava. A Course on Mathematical Logic. Springer, 1st edition, 2008.