Random Variables - Variance

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26 January, 2012

Outline

1. Recap

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2. Variance

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Recap

Main points

Random Variable,

Recap

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Random Variable, Distribution of a random variable (pmf),

Recap

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Random Variable, Distribution of a random variable (pmf), Expected Value.

Variance

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Simplification

$\mathbf{Cov}(X, Y) = E[X \cdot Y] - E[X] \cdot E[Y]$

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4. $\mathbf{Cov}(X, Y + Z) = \mathbf{Cov}(X, Y) + \mathbf{Cov}(X, Z)$.
5. $\mathbf{Var}(\sum_{i=1}^{n}(X_i)) = \sum_{i=1}^{n} \mathbf{Var}(X_i)$, if $X_1, X_2, \ldots X_n$ are **independent** random variables.

Bernoulli Variable

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$$p(0) = (1 - p)$$

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Observe that if X is a binomially distributed random variable with parameters n and p, then it can be expressed as a sum of n independent Bernoulli variables, i.e., $X = \sum_{i=1}^{n} X_i$, where each X_i is a Bernoulli random variable with parameter p.

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$$\mathbf{Var}(X) = n \cdot p \cdot (1 - p).$$

Geometric Variable

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If X is a geometric random variable with parameter p,

$$\mathbf{Var}(X) = \frac{1-p}{p^2}$$