Randomized Algorithms - Course Structure

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West Virginia University

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Outline

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Motivation

Why Randomized Algorithms

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(i) What is an algorithm?

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(i) What is an algorithm?

(ii) What resources do we analyze?

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Why Randomized Algorithms

(i) What is an algorithm?

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(iii) What is a randomized algorithm?

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Why Randomized Algorithms

(i) What is an algorithm?

(ii) What resources do we analyze?

(iii) What is a randomized algorithm?

(iv) What do we analyze?

Motivation

Why Randomized Algorithms

(i) What is an algorithm?

(ii) What resources do we analyze?

(iii) What is a randomized algorithm?

(iv) What do we analyze?

(v) Why are randomized algorithms preferred over deterministic ones?

Course Structure

Flow

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Flow

(i) Fundamentals and Applications.

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(i) Fundamentals and Applications.

(ii) High Probability Analysis.

Course Structure

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(i) Fundamentals and Applications.

(ii) High Probability Analysis.

(iii) Advanced Analysis Tools.

Course Structure

Flow

(i) Fundamentals and Applications.

(ii) High Probability Analysis.

(iii) Advanced Analysis Tools.

(iv) Advanced Applications (Seminar).

Prerequisites

Topics

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(i) Discrete mathematics, including mathematical induction and counting.

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(ii) Probability fundamentals, including measures of aggregation and dispersion.

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(i) Discrete mathematics, including mathematical induction and counting.

(ii) Probability fundamentals, including measures of aggregation and dispersion.

(iii) Design and analysis of algorithms for combinatorial problems.

Expectations

Guidelines

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Guidelines

1. Attendance.

Expectations

Guidelines

1. Attendance.
2. Inquisitiveness.

Expectations

Guidelines

1. Attendance.
2. Inquisitiveness.
3. Systematic hard work.

Expectations

Guidelines

1. Attendance.
2. Inquisitiveness.
3. Systematic hard work.
4. Perseverance.

Outline

General Information

Logistics

1. Semester - Spring 2012.
2. Meeting Times: Tu-Th, 09 : 30 am -10 : 45 am.
   Location: 355 ESB-E.
3. Contact Information: 749 ESB, ksmani@csee.wvu.edu.
4. Office Hours: By appointment.
5. Prerequisites: Exposure to probability and algorithm design.
6. Textbook - [MU05] is the main text, although [MR95] and [Ros00] are strongly recommended for supplementary reading.
7. URL - `http://www.csee.wvu.edu/~ksmani/courses/sp12/rand/rand.html`.

General Information (contd.)

Assessment

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Assessment

1. Homeworks (3) - There will be three homework assignments; each assignment is worth 20% of your grade.
   Table (1) details the homework schedule.

Assignment Date	Submission Date
01/27	02/02
03/01	03/08
04/03	04/10

Table: Homework Schedule

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Table: Homework Schedule

2. Presentation (1) - You will be required to present one topic which will be decided through discussions with the instructor. The presentation is worth 20 points.
3. Research - You are expected to engage in independent research on a problem of your choosing, related to the topics of this course. This research is to be summarized in a report, to be handed in on the last day of class and is worth 20 points. *Alternatively, you will be handed a fourth homework assignment, on May* 1*, which you will need to turn in by May* 8*.*

General Information (contd.)

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4. Final - There will be no final exam.

General Information (contd.)

Assessment

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Assessment

1. A maximum of 5 bonus points will be awarded for class performance.

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2. Grade Boundaries:

Grade	Boundary
A	80 and up
B	65 – 79
C	50 – 64
D	45 – 49
F	0 – 44

Table: Grade Boundaries

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3. Grading policy - If you have any questions about the grading, you must contact the intructor within two days of your paper being returned.

General Information (contd.)

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3. Grading policy - If you have any questions about the grading, you must contact the intructor within two days of your paper being returned.
4. Makeup Policy - If for some reason, you are unable to attend a test or an exam, please meet me at the earliest and I will set an alternate date.

General Information (contd.)

Course Objectives

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(a) Introducing the fundamentals of probabilistic analysis.

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(a) Introducing the fundamentals of probabilistic analysis.

(b) Analyzing algorithms in the probabilistic framework.

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(e) Introducing Randomized Complexity classes.

General Information (contd.)

Course Objectives

(a) Introducing the fundamentals of probabilistic analysis.
(b) Analyzing algorithms in the probabilistic framework.
(c) Introducing high probability analysis.
(d) Introducing the Monte Carlo method.
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Learning Outcomes

General Information (contd.)

Course Objectives

(a) Introducing the fundamentals of probabilistic analysis.
(b) Analyzing algorithms in the probabilistic framework.
(c) Introducing high probability analysis.
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Learning Outcomes

(a) Appreciate the fundamentals of randomized algorithm design.

General Information (contd.)

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(c) Introducing high probability analysis.
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Learning Outcomes

(a) Appreciate the fundamentals of randomized algorithm design.
(b) Develop randomized algorithms for variants of problems in $\mathtt{P}$.

General Information (contd.)

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(a) Introducing the fundamentals of probabilistic analysis.
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(d) Introducing the Monte Carlo method.
(e) Introducing Randomized Complexity classes.

Learning Outcomes

(a) Appreciate the fundamentals of randomized algorithm design.
(b) Develop randomized algorithms for variants of problems in $\mathtt{P}$.
(c) Apply high probability analysis to selected randomized algorithms.

General Information (contd.)

Course Objectives

(a) Introducing the fundamentals of probabilistic analysis.
(b) Analyzing algorithms in the probabilistic framework.
(c) Introducing high probability analysis.
(d) Introducing the Monte Carlo method.
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Learning Outcomes

(a) Appreciate the fundamentals of randomized algorithm design.
(b) Develop randomized algorithms for variants of problems in $\mathtt{P}$.
(c) Apply high probability analysis to selected randomized algorithms.
(d) Apply the Probabilistic Method for establishing the existence of properties in combinatorial structures.

General Information (contd.)

Course Objectives

(a) Introducing the fundamentals of probabilistic analysis.
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(d) Introducing the Monte Carlo method.
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Learning Outcomes

(a) Appreciate the fundamentals of randomized algorithm design.
(b) Develop randomized algorithms for variants of problems in $\mathtt{P}$.
(c) Apply high probability analysis to selected randomized algorithms.
(d) Apply the Probabilistic Method for establishing the existence of properties in combinatorial structures.
(e) Understand the fundamentals of Markov chains and the Monte Carlo method.

Outline

Topics to be covered

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Probability Fundamentals

Sample Space, Events, Assigning probabilities to events, Conditional probability, Independent events, Useful identities, Random Variables, Linearity of Expectation.

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A sampling of Randomized Techniques

Verifying polynomial identities, verifying matrix multiplication, A min-cut algorithm, The coupon collector's problem, Quicksort.

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Markov's inequality, Moments of a Random Variable, Chebyshev's inequality, Applications.

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Moments and Deviations

Markov's inequality, Moments of a Random Variable, Chebyshev's inequality, Applications.

The Chernoff Bound

Moment Generating Functions, Deriving and Applying Chernoff bounds, Special cases, Applications.

Topics to be covered (contd.)

Balls, Bins and Random Graphs

The Birthday Paradox, Balls into Bins, The Poisson Distribution, The Poisson Approximation, Applications, Random Graphs.

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The Probabilistic Method

The basic counting argument, The Expectation argument, Derandomization using conditional expectation, Sample and Modify, The Second Moment method, The Conditional Expectation Inequality, The Lovasz Local Lemma, Explicit Constructions using the Local Lemma, The general case of the Local Lemma.

Topics to be covered (contd.)

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Markov Chains and Random Walks

Definitions and Representations, Classification of states, Stationary distributions, Random walks on undirected graphs, Parrondo's paradox.

Topics to be covered (contd.)

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The Monte Carlo Method

The Monte Carlo Methods, The DNF counting problem, From Approximate sampling to Approximate counting, The Markov Chain Monte Carlo Method.

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Martingales

Martingales, Stopping Times, Wald's Equation, Tail Inequalities for Martingales, Applications.

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Social Justice Statement

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West Virginia University is committed to social justice. I concur with that commitment and expect to foster a nurturing learning environement, based upon open communication, mutual respect and non-discrimination. Our University does not discriminate on the basis of race, sex, age, disability, veteran status, religon, sexual orientation, color or national origin. Any suggestions to further such a positive and open environment in this class will be appreciated and given serious consideration. If you are a person with a disability and anticipate needing any type of accommodation in order to participate in this class, please advise me of the same and make appropriate arrangments with Disability Services ($293 - 6700$).

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Bibliography


Rajeev Motwani and Prabhakar Raghavan.
Randomized Algorithms.
Cambridge University Press, Cambridge, England, June 1995.


Michael Mitzenmacher and Eli Upfal.
Probability and Computing.
Cambridge University Press, 1^{st} edition, 2005.


Sheldon M. Ross.
Probability Models.
Academic Press, Inc., 7th edition, 2000.