Randomized Algorithms - Scrimmage I

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1 Instructions

1. The scrimmage will not be graded, although bonus points could be awarded depending upon performance.
2. You are encouraged to solve as many problems as you can.
3. Solutions will **not** be posted, although you may contact the instructor to discuss your approaches.

2 Problems

1. A laboratory test is 95% effective in detecting a disease when it is in fact, present. Unfortunately, the test also yields a "false positive" in 1% of the healthy persons tested. If 0.5% of the population actually has the disease, what is the probability that a person has the disease, given that his test result is positive?
2. A man has two coins in his pocket; a two-headed coin and a fair coin. He choses one of the two coins uniformly and at random. He then tosses the coin and it shows heads. What is the probability that he chose the fair coin. Assume that he flips the coin again and it shows heads again. What is the probability that he chose the fair coin? Just for kicks, he flips the coin a third time and it shows up tails. What is the probability that he chose the fair coin?
3. Let X and Y denote random variables over some probability space. Let c denote a constant and let $g(X)$ denote some function of X. Argue that:
   (i) $E[g(X)] = \sum_x g(x) \cdot p(x)$, where $p()$ is the probability mass function (pmf) of X.
   (ii) $E[X^2] \geq (E[X])^2$.
   (iii) $E[X + Y] = E[X] + E[Y]$.
   (iv) $E[c \cdot X] = c \cdot E[X]$.
   (v) $\mathbf{Var}(c \cdot X) = c^2 \cdot \mathbf{Var}(X)$.
4. A random variable X is said to be uniformly distributed over the interval (a, b), if it has the following probability mass function:
$$p(x) = \begin{cases} 1, & \text{if } a < x < b \\ 0, & \text{otherwise} \end{cases}$$
   Compute $E[X]$. Suppose the length of a square is chosen uniformly from the interval $[1, 99]$. What is the expected area of the square?
5. Let X denote a geometric random variable with parameter p. Compute $E[X]$, using the method of conditional expectation.