Randomized Algorithms - Scrimmage II

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1 Instructions

1. The scrimmage will not be graded, although bonus points could be awarded depending upon performance.

2. You are encouraged to solve as many problems as you can.

3. Solutions will **not** be posted, although you may contact the instructor to discuss your approaches.

2 Problems

1. After lunch one day, Alice suggests to Bob the following method to determine who pays. Alice pulls three six-sided dice from her pocket. These dice are not the standard dice, but have the following numbers on their faces:

   (a) Die $A : 1, 1, 6, 6, 8, 8$.
   (b) Die $B : 2, 2, 4, 4, 9, 9$.
   (c) Die $C : 3, 3, 5, 5, 7, 7$.

   The dice are fair, so each side comes up with equal probability. Alice explains that she and Bob will each pick up one of the dice. They will each roll their die, and the one who rolls the lowest number loses and will buy lunch. So as to take no advantage, Alice offers Bob the first choice of the dice. Argue that regardless of which die Bob chooses, Alice has a winning die?

2. Let X and Y be independent geometric random variables, where X has parameter p and Y has parameter q.

   (a) What is the probability that $X = Y$?
   (b) What is $\mathbf{E}[\max(X, Y)]$?
   (c) What is $P(\min(X, Y) = k)$?
   (d) What is $\mathbf{E}[X|X \leq Y]$?

3. If $X \sim B(n, \frac{1}{2})$, argue that the probability that X is even, is one-half.

4. Roll a standard, fair die over and over till you get the first pair of consecutive sixes. What is the expected number of rolls?