Combinatorial Optimization

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1. Weak duality theorem.
2. Strong duality theorem.
3. Slack and Surplus variables.
4. Weak complementary slackness theorem.
5. Strong complementary slackness theorem.
6. Complementary slackness for canonical form. Let $\mathbf{s} = \mathbf{b} - \mathbf{A} \cdot \mathbf{x}$ and $\mathbf{t} = \mathbf{y} \cdot \mathbf{A} - \mathbf{c}$. If $\mathbf{x}^*$ and $\mathbf{y}^*$ are the primal and dual optimal, then, $x_j * \cdot t_j^* = 0$, $\forall j = 1, 2, \ldots n$ and $s_i * \cdot u_i^* = 0$, $\forall i = 1, 2, \ldots m$.

   Proof:

$$\begin{aligned} \mathbf{c} \cdot \mathbf{x}^* &= (\mathbf{y}^* \cdot \mathbf{A} - \mathbf{t}^*) \cdot \mathbf{x}^* \\ &= \mathbf{y}^* \cdot \mathbf{A} \cdot \mathbf{x}^* - \mathbf{t}^* \cdot \mathbf{x}^* \\ &= \mathbf{y}^* \cdot (\mathbf{b} - \mathbf{s}^*) - \mathbf{t}^* \cdot \mathbf{x}^* \\ &= \mathbf{y}^* \cdot \mathbf{b} - \mathbf{y}^* \cdot \mathbf{s}^* - \mathbf{t}^* \cdot \mathbf{y}^* \\ \Rightarrow \mathbf{y}^* \cdot \mathbf{s}^* + \mathbf{t}^* \cdot \mathbf{y}^* &= 0 \end{aligned}$$

   □

7. Graphical Solution.
8. The Simplex method (Samir's notes).