First Order Theories - Arrays

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Outline

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The theory of arrays can be used to reason about programming languages which use array structures.

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(ii) $a\langle i \triangleleft v\rangle$ (**write**) is a ternary function: $a\langle i \triangleleft v\rangle$ represents the modified array a, in which position i has value v.

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(iii) $=$ is a binary predicate.

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($\mathcal{A}4$.) $(\forall a)(\forall i)(\forall j)\ i \neq j \rightarrow a\langle i \triangleleft v\rangle[j] = a[j]$.

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