First Order Theories - Combination Theories

K. Subramani¹

¹Lane Department of Computer Science and Electrical Engineering West Virginia University

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Outline



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1 Introduction

Combination Theories

Main Ideas

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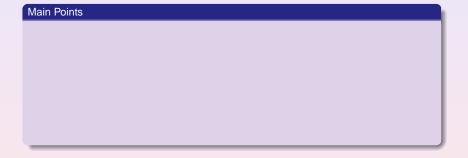
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satisfiability in the quantifier-free fragment of the combination theory $T=T_1\cup T_2$ is decidable. Furthermore, if the decision procedures for T_1 and T_2 are in \mathbf{P} , then so is the combined decision procedure for $T_1\cup T_2$.



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- Ocnsider the formula $H: f(f(x) f(y)) \neq f(z) \land x \leq y \land (y + z) \leq x \land 0 \leq z$. Is $H(T_E \cup T_0)$ -satisfiable?