First Order Theories - The Theory of Equality

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Outline

1. Equality

The Theory of Equality

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(i) *A axiom* **schema** *stands for a set of axioms, each an instantiation of the parameters.*

(ii) *The theory of equality is undecidable.*

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Is the formula $F\ :\ (a = b) \wedge (b = c) \rightarrow g(f(a), b) = g(f(c), a)$ valid?